THE CLINICAL USE OF

PRISMS

MADDOX

SECOND EDITION

OPHTHALMOLOGICAL PRISMS.

THE CLINICAL USE OF PRISMS;

AND THE

Decentering of Lenses.

BY

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PREFACE

TO THE SECOND EDITION.

A CONSIDERABLE amount of new material has been added to this edition, consisting partly of a fuller study of convergence, and partly of a number of optical problems and practical points worked out since the first edition. The number of diagrams, too, has been considerably increased. It has been endeavoured to maintain the utmost simplicity in the optics of the subject, and while giving enough to afford interest and precision, preference has been given to graphic or mechanical demonstrations, so that no unmathematical reader will, it is hoped, experience any difficulty, unless perhaps with two or three formulæ, which can be neglected without loss. My thanks are due to Mr. Berry for being so kind as to

read the proof sheets, and for valued comments thereon. Though the undoubted value of strong prisms is confined to a rather limited sphere, a knowledge of the principles of convergence, and the decentering of lenses is advantageous to all students of refraction.

20, ALVA STREET,
EDINBURGH,
July, *1893*.

EXTRACT FROM PREFACE TO FIRST EDITION.

The title of this little work indicates its chief features, but does not cover its ground completely. Its first object was to communicate a series of aids to precision in the use of prisms, worked out during several years, which it is hoped will be of some service in this difficult by-way of ophthalmic practice. They have, however, been introduced by a sketch of the simplest properties of prisms, and supplemented by a brief account of their chief clinical uses. . . . . It will not be supposed that the precision aimed at in these pages is necessary in the majority of refraction cases. It is chiefly anomalies of convergence, faulty tendencies of the ocular muscles, and the needs of the increasing neurasthenic class of patients that are kept in

view. In these last cases, spectacles cause discomfort, however perfectly refraction may be corrected, unless the lenses are also suitably placed in respect of convergence, and sometimes even then, for a time.

Part of the manuscript was kindly read over by my friend, Dr. Poulett Wells, to whom I am indebted for one or two valuable corrections, and, indeed, for suggesting the need of a book on prisms.

July, 1889.

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CHAPTER XVI.

PRISMS.

Chapter I.

GEOMETRICAL PROPERTIES OF PRISMS.

OPHTHALMOLOGICAL Prisms are simply wedges of transparent material, contained between two plane faces, which are inclined to one another at a moderate angle. The "strength" of a prism depends on the size of this angle, which varies therefore in different prisms, and also on the refractive properties of the material from which the prism is made.

Since the terminology of prisms in optics, and especially in ophthalmology, is so different from that of prisms in geometry and physics, it will be well to notice in some detail the terms employed.

Refracting Surfaces.—By "refraction" is meant the deflection, or sudden bending from its former course, of a ray of light as it enters (other than perpendicularly) the surface of a rarer or denser medium. The body of a prism does not refract light, since rays travel through its substance in straight lines. It is only as they enter or leave the prism, and thus pass from one medium to another that they undergo refraction. It is for

this reason that the two plane polished faces are called the "refracting surfaces." In *Fig.* 1 they are represented by the faces *a c f b* and *a d e b.*

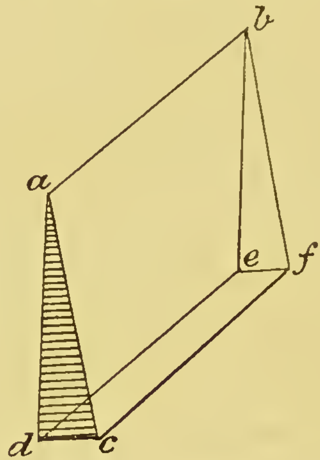

Fig. 1.—A Prism.

Angle of Refraction.—We have seen that the two plane faces of a prism are made to slope towards each other so as to contain between them a moderate angle. This angle is called the "angle of refraction," because the total effect of the prism on the direction of a ray of light depends on the size of this angle, as well as on the refracting power of the material from which the prism is made. Until recently, clinical prisms have had the number of this angle in degrees scratched upon them, so that prisms marked, *e.g.*, with 1° or 2° were known to have their refracting surfaces inclined to each other by 1° or 2° respectively. Trial cases used generally to be provided with a series of prisms,

marked in this way, from 1° or 2° up to 12° or 24°. The angle of refraction is the one item the optician needs to know in order to grind a prism. He first sets a pair of compasses so that their legs shall enclose the prescribed angle, and then cuts and grinds the piece of glass to the shape of a wedge that will just fit in between the two legs. To measure the refracting angle of a prism, or, as we

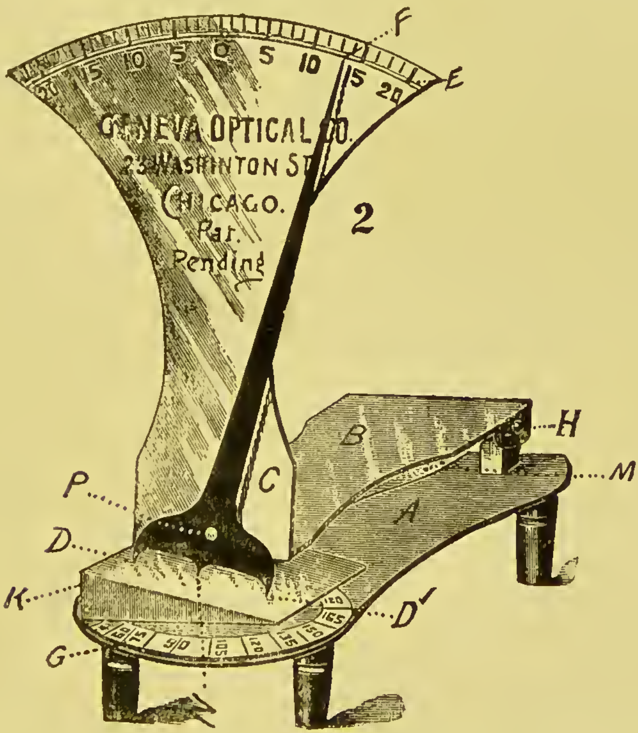

Fig. 2.—Prism Measure: the index points to the required angle when the teeth at its foot are pressed upon the prism.

may simply call it, "*the* angle" of a prism, we may resort to the same manœuvre, and after grasping the prism between a pair of compasses, measure

the angle they include on a protractor.* But if the prism be mounted in a metal ring, this method is impracticable, and the "prism measure" of the Geneva Optical Company will be found useful *(Fig. 2)*.

What, however, is of interest to the surgeon, is the optical power of a prism, and not its mere physical conformation, and it is undoubtedly better for trial prisms to be marked by their deviating power over a beam of light, as described later on.

PRINCIPAL SECTION.—Any section made through a prism at right angles to its refracting surfaces is called a "principal section."† Such a section is shown in *Fig.* 3. The two refracting surfaces are

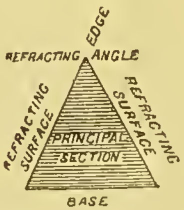

Fig. 3.—The same prism in section.

seen to meet at the edge of the prism, enclosing between them the "refracting angle." The third side, opposite the edge, is called the base.

RECTANGULAR PRISMS.—In geometry, when we

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* For a diagram of a protractor, put to a different use, see *Fig.* 13.

† To put it another way, it is any section made perpendicularly to the axis of the prism.

speak of a rectangular prism, we mean one which has a rectangular section. Ophthalmological prisms, however, are always triangular in section, as shown in *Fig.* 3, so that when we speak of them as "square," "rectangular," "circular," and so forth, it must be understood that these expressions only apply to the shape of the refracting surfaces. It is evident that two plane surfaces may be of most various shapes, and yet be inclined to each other by the same angle, so that what we call the "shape" of a prism, in no way affects its strength. A rectangular prism is shown in *Fig.* 4, of a kind not

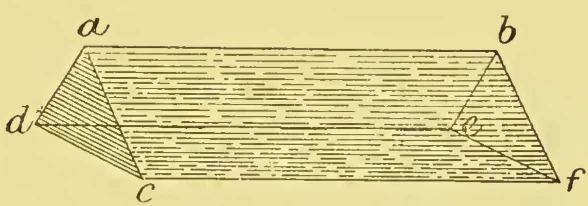

Fig. 4.—A rectangular prism.

used clinically, but rather by students of optics. A *square* prism, which is of course a variety of the rectangular, is represented in *Fig.* 1. In both figures the "refracting surfaces" are lettered *a c f b* and *a d e b*, meeting along the "edge" *a b*, and enclosing the "refracting angle" *d a c*, which is seen to be smaller in *Fig.* 1 than in *Fig.* 4. The remaining surface *d e f c* is called the "base."

For some clinical purposes square prisms are

extremely useful, since it is so easy to hold them in any desired position. Their disadvantage is that they will not fit into the ordinary trial-frame, but in every other particular this shape is preferable. It would be well for trial-cases to be provided with at least one square prism, having a refracting angle of about 12°. A convenient way of using square prisms is that recommended by Noyes, in which a series of increasing strength are joined together in a frame, so that one after another can be made to pass before the eye. Such a series is shown in *Fig.* 5.

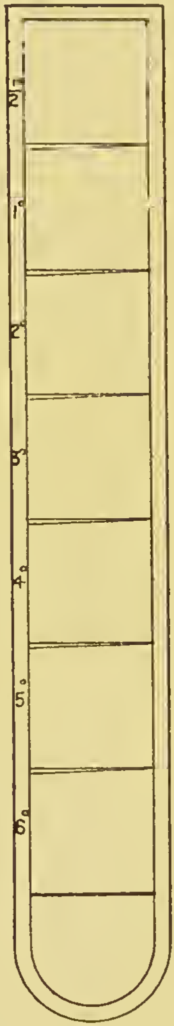

Fig. 5.—A "bar of prisms" (Gen. Opt. Co.).

CIRCULAR PRISMS.—Clinical prisms are almost universally circular, to admit of their insertion into the same trial-frame as clinical lenses.

One is shown in *Fig.* 6, and the hypothetical design

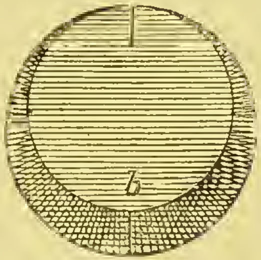

Fig. 6.—A circular prism.

in *Fig.* 7 is intended to show how a circular prism is related to a rectangular one, as though it were cut out of it.

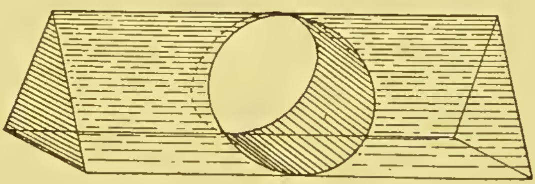

Fig. 7.—A hypothetical illustration to show the relation of a circular prism to a rectangular one.

Apex and Base.—The thinnest part of a circular prism is called the *apex.* It is the point where the two refracting surfaces meet, or most nearly meet. The *base* of a circular prism is the thickest part of it, exactly opposite the apex. It is not therefore a surface as in the case of a rectangular prism, but an imaginary line (*b* in *Fig.* 6) uniting the most separate points of the two circular surfaces. An imaginary straight line from the apex to the centre of the base we may call, for convenience, the *base-*

apex line. All objects viewed through prisms appear displaced in a direction parallel to the base-apex line, as shown in *Fig.* 8, where a real object *O* appears displaced to *e* in a direction parallel to the base-apex line *a f.*

Edge of a Prism.—It is easy to understand what is the edge of a rectangular prism, for, as already explained, and shown in *Figs.* 4 and 7, it is the line *a b*, in which the refracting surfaces meet. But in a circular prism the surfaces meet only at a point. A circular prism therefore has no actual edge, but only an imaginary one, viz., an imaginary line which passes through the apex at right angles to the base-apex line, as *b c* in *Fig.* 8. It is the

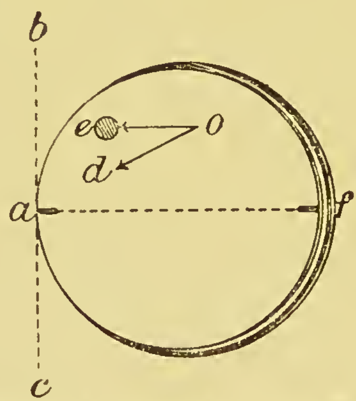

Fig. 8.—To show how an object at O appears displaced, not in the direction *d* towards the apex *a*, but in the direction *e* towards the edge *b c*, and parallel to the "base-apex line," *a f.*

line in which the two refracting surfaces *would* meet if they were prolonged, and it coincides therefore with the edge of the rectangular prism, out of which we may imagine the circular prism to have been

cut. The name is taken from the edge of a knife in which the two polished faces of the blade meet. Though the edge of a circular prism has no real existence, it has a very real importance, for all objects seen through prisms appear displaced towards their *edge.*

CHAPTER II.

SIMPLEST OPTICAL PROPERTIES OF PRISMS.

REFRACTION BY PRISMS.—In the case of a well-made clinical prism, we have only two media to take account of (both of which are practically homogeneous and isotropic), viz., air and glass. A homogeneous medium is one of equal density throughout. An isotropic medium is one that is singly refracting, that is, when a ray of light enters the surface of an isotropic medium, it continues as a single ray after refraction.

A doubly refracting medium possesses certain surfaces which have the property of dividing a ray of light into two rays, one being refracted in a different direction to the other. Rock crystal is an instance, and though it is possible to cut weak prisms out of this material in such a way that the refracting surfaces shall not, to an appreciable extent, refract doubly, crown glass is more desirable. Different media vary greatly in density, and the denser a medium, the more it retards the velocity of light. Thus when a ray of light, travelling in air, reaches the surface of a piece of glass,

its velocity becomes instantly reduced by more than one-third, and after travelling through the glass at this lessened speed, it regains its former velocity as it again emerges into air.

A consequence of this remarkable property of media is, that whenever a ray of light enters a medium of different density, other than perpendicularly, it is bent or refracted from its former course. If the medium be homogeneous, the ray pursues a perfectly straight course through it, and is only refracted at its surfaces. To show how this refraction takes place, it will be necessary to touch lightly on some of the elementary properties of light, which, according to the undulatory theory, consists of vibrations propagated in a hypothetical substance, called "æther," which is supposed to permeate the whole of space. These vibrations occur transversely, or at right angles to the axis of propagation of the ray of light: just as when a stone is flung into a pond, though the waves travel rapidly towards the banks of the pond, the actual particles of water do not move towards the banks at all, but only rise and fall, mounting to the summit of a wave, and then sinking into the trough left behind it, so that the motion of the particles is at right angles to the direction in which the waves are propagated. It is the same with light, only with these differences, that in æther the tiny side-to-side movements are not in one direction only, like the

perpendicular* movements of the water (except in polarised light), but in all directions at right angles to the axis of the ray. Moreover, while the waves in a pond travel along the surface only, light travels in all directions from its source, so that we have to do with three dimensions instead of two. When light is generated at a point, it tends to travel outwards from that point, not as we have seen, in an ever-widening *circle*, like the waves of a pond, but in an ever-increasing *sphere* (unless hindered, as it always is, in some directions by bodies in its way which absorb, refract, or reflect it). The waves in a pond are enlarging circles, those of light form enlarging spheres, and in each we can speak of a *wave-front*. The motion of the waves is always in a direction perpendicular to the wave-front,† a fact which, if grasped, will help us much. A moment's reflection will remind us that this is true

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* It is true that the motion of the particles of water is not quite so simple as this, since they describe curves, sometimes even ellipses or circles, which, being parallel to the direction of propagation, spoil the illustration for light, where the circular or elliptical motions of the particles of æther are perpendicular to the direction of propagation.

† I do not trouble the reader by explaining *why* this is so. It really depends on the principle of Huygens, that the disturbance at any point of a wave-front is the resultant of the separate disturbances which the different portions of the same wave-front, in any one of its earlier positions, would have occasioned if acting singly.

in the waves of water. Imaginary lines, drawn in all directions from the spot where the stone entered the water, will be radii of the circles, and represent the directions in which the waves advance. Now, if the pond be perfectly circular, the waves generated by a stone dropped exactly into its centre will break upon the shore with the direction of their wave-fronts unaltered. If, however, the pond is not circular, wherever the waves meet the shore obliquely, the direction of the wave-fronts will be altered, as they break upon it. Similarly with light; when it falls perpendicularly on the surface of a new medium, the direction of its wave-front is unchanged, but if it falls obliquely, the wave-front is altered in direction. There is this difference, however, that the light waves are not like those of the water, broken upon the shore, but go on travelling through the medium in a new direction—a direction perpendicular to the *new wave-front.* Let us now ask, "How is it that the surface of a new medium can alter the direction of the wave-fronts that meet it obliquely?" It depends on the fact already mentioned, that light does not travel with equal velocity through different media, but that its velocity is retarded in proportion to the density of the medium: just as a column of soldiers, crossing a desert at "quick march," when they come to a canal that lies across their path, have their velocity retarded, by progression through water not being so rapid as

progression through air. If the canal lie straight across their path, so that all the men in the front rank reach the bank and enter the water simultaneously, the column has the direction of its front unchanged. When light comes from a considerable distance, the wave-fronts may be regarded as linear, so that the ranks of soldiers represent successive wave-fronts.

If, as in *Fig.* 9, the same column of soldiers find a canal crossing their path *obliquely*, one man in the

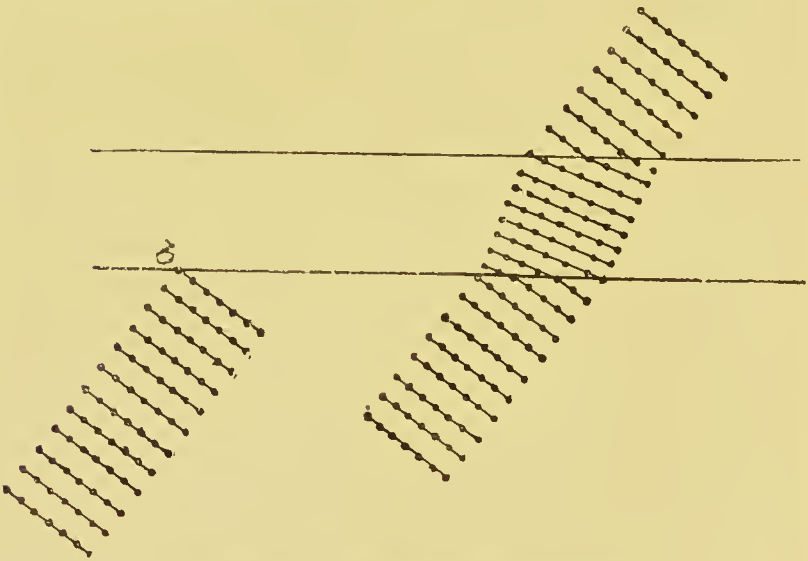

Fig. 9.—To illustrate refraction of light.

front rank (*b*) reaches the water before his comrades in rank with him, and since he is retarded before they are, they gain on him, and the direction of the front of the column is altered. The same may be said of all the succeeding ranks, and it

illustrates how the successive wave-fronts of light as they break obliquely on the surface of a denser medium, become altered in direction. The same man, however (*b*), who was first to enter the water, will also be first to get out of it, and he will then gain on his comrades (who are still in it) as much as he lost before, so that the column resumes its original frontage, and pursues its original direction, being only displaced a little to one side, as shown by the figure. If, on the other hand, the canal were triangular in shape, instead of having parallel banks, the original direction would not be resumed, for the man who first entered the water might have to leave it last, or, even if he left it first, he would

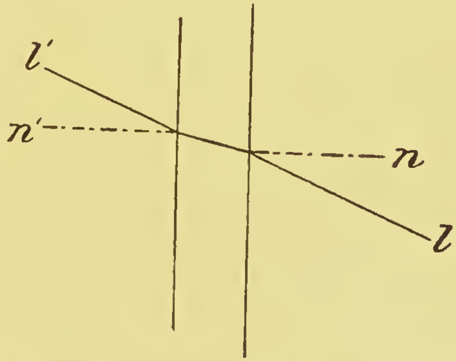

Fig. 10.—Refraction by a plate of glass.

not gain, on leaving it, as much as he lost on entering it. This is analogous to what happens in the case of prisms, for we have seen they are triangular in section. While the refracting surfaces of a pane of glass are parallel, as in *Fig.* 10 those of a prism

are inclined to each other. The consequence is, that when rays of light traverse a prism, they are bent from their previous course, and emerge in a new direction. An imaginary line, perpendicular to the surface of a medium, is called a "normal" to that surface. Thus in *Fig.* 10, which represents a pane of glass traversed by a ray of light $l\,l'$, the dotted line n is the normal at the point of incidence of the ray, and the line n' is the normal at its point of emergence. It will be seen that on entering the denser medium, the ray is bent (or "refracted") *towards* the normal (n), while at its emergence from the denser medium it is refracted *away* from the normal (n').

DEVIATION.—The deviation from its former course which a ray of light suffers by traversing a prism is the combined result of the two refractions which it has undergone at the two surfaces. The denser the glass, and the greater the angle of the prism, and the more obliquely it is traversed by the ray, the greater is the total deviation it produces. With prisms of ordinary crown glass, the "angle of deviation" for a ray of light passing through the prism (in the direction of "minimum deviation," p. 24) is slightly more than half the refracting angle of the prism, if it be a weak prism, and increasingly more than half as the refracting angle increases. In *Fig.* 11 a ray (ef) is seen traversing a prism. On entering the glass

it is bent towards the normal (n), and on leaving it away from the normal (n'), so that its course is altered by its passage through the prism; the ray (e), incident on the first surface, was pursuing a course towards p, but it now proceeds towards f.

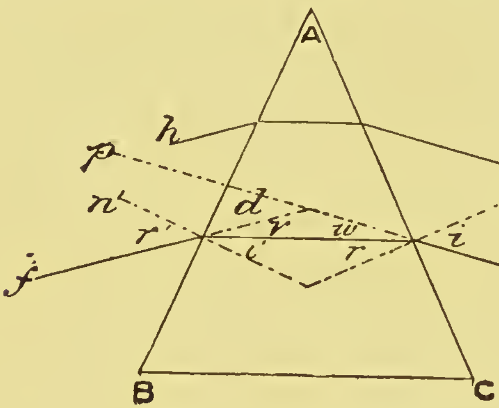

Fig. 11.—Refraction by a prism.

The angle d contained between the forward prolongation of the entering ray (e), and the backward prolongation of the emergent ray (f) is the *deviating angle*, or, shortly, the "*deviation*" of the prism. The amount of deviation suffered by a ray is the same, whatever part of the prism it traverses, provided it forms the same angle with the normal at the first surface. Thus parallel rays as g, e, in *Fig.* 11, also emerge parallel. In the figure, the angle of deviation at the first surface is w, and that at the second surface is v, and it is evident at a glance that the total deviation at the prism is the sum of these two deviations. The figure is so

constructed that the angle of incidence at the first surface (i), and the angle of refraction at the second (r') are equal, and in that case the deviations at the two surfaces are equal, or $v = w$. When we speak of the deviating angle of a prism, we assume the deviation to be that of a ray subject to these conditions, for, as we shall see, under other conditions the deviation is greater.

Fig. 12 shews, in a very simple, yet quite exact

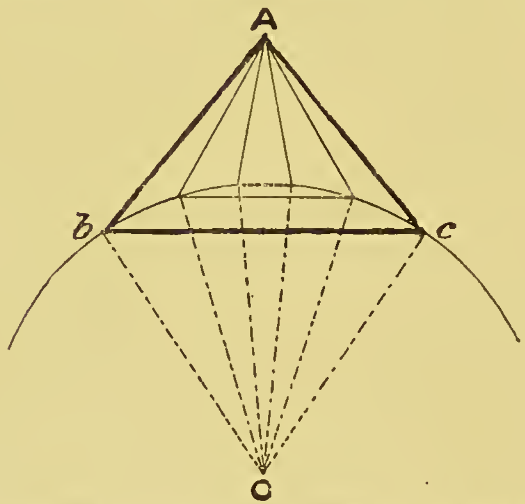

Fig. 12.—To shew the proportion between the refracting angles (A), and the deviating angles (O) of prisms.

way, how the deviating angles of prisms increase more rapidly than their refracting angles. Three prisms are delineated, in principal section, with their apices meeting at A. The dotted lines from the base of each prism enclose between them the deviating angle of that prism. A prism only a trifle stronger than the largest in the figure would

have a deviating angle *equal* to its refracting angle. But since for clinical purposes we never approach anything like this strength, and the smallest prism in the figure represents about the largest we prescribe, we have never to do with a deviating angle much more than half the refracting angle. The same figure shews us how easily we may find the deviating angle of any prism by construction. Draw any circle about a point *O*, and take any point *A* outside the circle, such that the distance *A O* shall be to the radius of the circle in the ratio of the refractive index of the glass. Thus, with crown glass, since its average refractive index is 1·54, if we make the radius one decimetre we shall make the distance *A O*=1·54 decimetres. Then with *A* as apex, draw the prism so that its base *b c* shall be a chord of the circle, and its refracting angle *A* shall be bisected by a straight line between *A* and *O*. Then lines drawn from the centre (*O*) to the extremities of the base, enclose the deviating angle required.

With weak prisms, there will be only a small error if we express the ratio of the deviating angle to the angle of refraction, by the decimal figures in the index of refraction of the glass. Thus, if the glass have an index of 1·5, the deviating angle is half (·5) of the refracting angle. If the index be the usual one of 1·54, the deviating angle is rather more than half (·54), and so on. For exact formulæ,

the reader is referred to Appendix. Dr. Jackson says that of late years the tendency has been to raise the index of crown glass, and that many years ago it was nearer 1·52. His proposal before the Ninth International Congress, to have all prisms marked by the deviation they produce instead of by their refracting angles, has been generally approved. The Committee, appointed to consider the proposal, recommended, in 1888, that :—

(1,) Prisms ought to be designated by the number of degrees, "minimum deviation," they produce.

(2,) Where intervals of less than one degree are desired, half degrees and quarter degrees should be used.

(3,) To indicate that degrees of deviation are meant, the letter *d* should be used; thus prism 2°*d* will indicate a prism that produces a minimum of two degrees.

The following table shews the angle of deviation for prisms of from 1° to 12° by the ordinary

TABLE I.

Angle of Refraction.	Angle of Deviation.	Angle of Refraction.	Angle of Deviation.
1°	32′	7°	3° 48′
2°	1° 5′	8°	4° 20′
3°	1° 38′	9°	4° 54′
4°	2° 10′	10°	5° 26′
5°	2° 42′	11°	5° 59′
6°	3° 15′	12°	6° 32′

marking, if the refractive index be taken as 1·54. It will be seen that with the highest of the series there is an error of only half a degree in assuming, as we are wont to do, the deviating angle to be half the refracting angle; this being an error of eight and a half per cent.

It must be confessed that the dioptric measurement of prisms lays a great burden on the optician, for whereas, formerly, he could test the correctness of his angle at all stages of his work, it is impossible to test the deviation of a beam of light until the polishing process has begun, so that the earlier processes have to be repeated. Perhaps the best plan is that recommended by Prentice—to keep a large number of prisms in stock, of all sizes, and select the one whose deviation is nearest the prescription.

In attempting to make a prism of definite dioptric power, an optician should have a good idea to start with of the angle which, when imparted to the glass, will be most likely to satisfy the requirements, and to do this he should multiply the prescribed angle by two, and subtract nine minutes for each degree in the prescribed angle if the prism is a weak one, or ten minutes for each degree if the prism is a strong one (over $6°d$).

Thus to produce a prism of $2°d$, he would cut the glass to an angle of $3° 42'$; that is, to $2 \times 2°$ *minus* $9 \times 2'$. To make a prism of $9°d$, he would

cut the glass to twice nine degrees, *minus* ten times nine minutes, or 16° 30′.

To make, however, calculation of any kind unnecessary, what we might call a *deviation protractor* could easily be constructed; so that the optician, instead of adjusting the legs of his compasses to an ordinary protractor (*see* p. 3) to get the refracting angle, might adjust them to the deviation protractor, knowing that a prism made to fit in between the legs would then produce the deviating angle. All that is necessary is to work, as it were, with an ordinary protractor upside down, and place the pivot of the compasses at a point whose distance from the centre of the arc is 1·54 times greater than the radius of the arc, as shewn greatly reduced in *Fig.* 13. The legs of the compasses should be

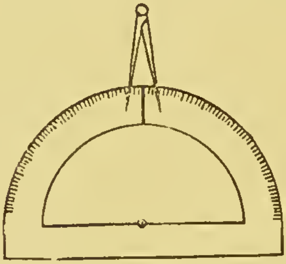

Fig. 13.—To shew how a protractor could be used to make prisms of definite deviations.

opened symmetrically with respect to a line joining their pivot and the centre of the arc. Or, on the same principle, a special instrument could be made, as in *Fig.* 14.

But as prisms are at present manufactured, it is extremely difficult to make them with mathematical accuracy.

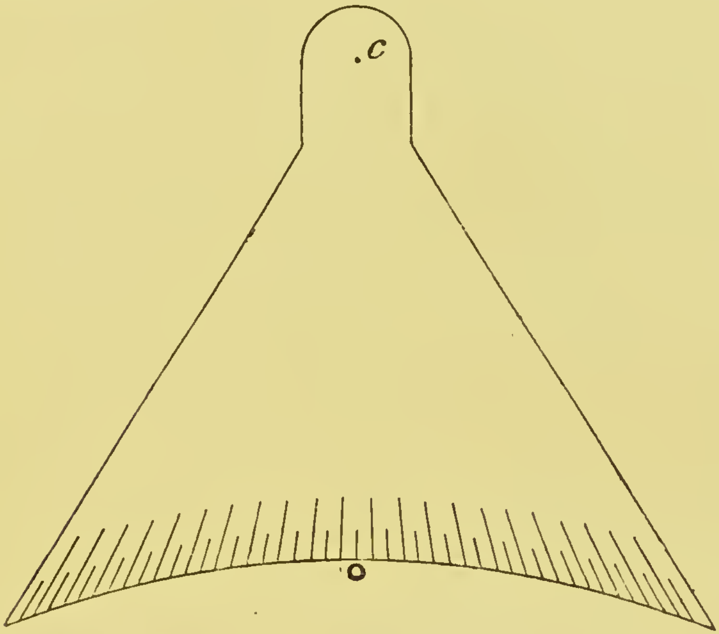

Fig. 14.—A proposed deviation protractor. The distance of *c* from the centre of the arc *o* is 1·54 times the radius of the latter. The arc is marked in ordinary degrees.

Duane has pointed out a disadvantage in the dioptric measurement of prisms in the hands of the oculist, just as the one last spoken of affects the optician. It is that if prisms be combined by apposition against one another, the deviation produced by the combination is not equal to the sum of their several deviations, and indeed it would be far easier to calculate precisely the effect of the combination if the refracting angles were known,

than if only the deviating angles were known. In this particular the disadvantage is undoubted, and "accordingly" he adds with truth, "in all problems involving the use of several prisms together, the old system is much more convenient than the new." *

Minimum Deviation.—When the points of incidence and emergence of a given ray are equi-distant from the edge of the prism, the ray is said to traverse the prism *symmetrically*, because the angles of incidence and emergence are then, as shewn in *Fig.* 11, equal. A ray under these conditions suffers less total deviation than if it passed through the prism in any other direction, and is therefore said to traverse the prism in the "direction of minimum deviation," or, to put it another way, the *prism* is said to be (in reference to the ray) in its "position of minimum deviation."

It is easily demonstrated by looking at a small object through a prism, and then rotating the prism about its axis,† when the apparent displacement of the object will increase with each increment of obliquity imparted to the prism. It will be found that there is a certain position which gives smaller appearance of displacement than any

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* "Archives of Ophthalmology;" Vol. XX., p. 321.

† The axis of a prism is an imaginary straight line parallel to the edge, and mid-way between the edge and the base.

other, and that when the prism is in this position small rotations of it have no appreciable effect on the amount of deviation. The position of minimum deviation is that most suitable for measuring the optical strength of prisms, because it is the position in which slight accidental aberrations have least effect.

The fact that a ray of light is more strongly deviated when it passes through a prism in one direction than in another, is an important one, accounting as it does for "prismatic astigmation," and for one or two varieties of the distortions produced by strong prisms. For these the reader is referred to p. 40. Meanwhile, a simple mechanical method I have adapted for comparing the angles of deviation with rays of varying incidence, may be described. Let two circles be described about a common centre *O* (*Fig.* 15), with their radii in the ratio of the refractive index. That is, if the radius of the smaller circle be 1, that of the larger will be 1·54 in the case of crown glass. At the extremity (*I*) of the smaller radius insert a pin perpendicularly into the paper, and after cutting a triangular piece of paper, *a b c*, to represent any given prism, lay it down with its side *a c* touching the pin (*I*), and its apex (*a*) touching the outer circle. The right hand edge *a b* of the movable triangle will then mark off on the smaller circle the arc of the angle of deviation, while the angle (*i*), between the

margin ($a\ c$) of the triangle and the line *Io* from the pin to the centre of the circle, represents the angle of incidence for each position.

By rotating the triangle into various positions, ever keeping the apex in contact with the outer circle, and the left hand margin in contact with

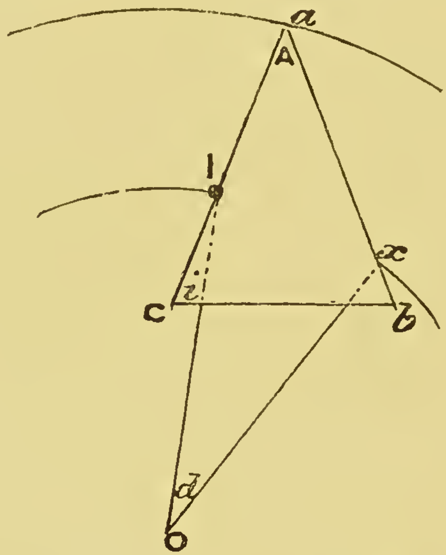

Fig. 15.—Mechanical method of shewing how the deviation produced by a prism alters with the angle of incidence.

the pin, we can find in a moment the arc of the deviation produced for any required angle of incidence; that is, for any value of *i* the deviation is represented by *d*. It will be noticed that the position of the paper triangle in which it marks off least of the smaller circle, and therefore indicates the smallest possible angle of deviation, is when the line which bisects its angle *A* passes through the

centre *O*. This is the "position of minimum deviation." It may be said, in passing, that the size of the angle of emergence is represented in every position by the angle between *a b* and *x o*.

It is sometimes desired to find the deviation suffered by a ray which enters one surface of a prism perpendicularly, and which, therefore, is only refracted at the second surface. Indeed, Prentice recommends this position for the measurement of prisms, as being most suitable for his "Prismometer." To find what is the deviation to be expected under these circumstances, we have only to revolve the paper prism till the margin *a c* coincides with the radius *I o*, when the other margin *a b* will mark off the arc of the deviation required.

DISPERSION. — By dispersion, or "chromatic aberration," is meant the breaking up of light into its constituent colours. It is a property possessed in greater or less degree by all prisms, since the differently coloured rays, being unequally refrangible, are deflected through different angles, the violet rays experiencing the greatest deflection and the red the least, while the paths of the rays of intermediate colours are spread out between these two extremes. The whole solar spectrum can thus be received upon a white screen arranged to intercept a pencil of light which has traversed a strong prism. This property of dispersion, so

valuable in spectrum analysis, is a serious disadvantage in the ophthalmological use of prisms, from the coloured margins it appears to give to objects, and the want of definition it occasions, just in proportion to the strength of the prism used.

It is only fairly weak prisms, therefore, that can be clinically employed with much advantage. It is not often advisable to exceed 3° or 4°, though the limits depend on the visual acuity, and habits of observation, of the patient. In amblyopia from corneal nebulæ, stronger prisms can be borne. The dispersive power of crown glass is so much less than of flint glass that it is invariably employed for clinical purposes. Achromatic prisms can be made by cementing together two prisms, one of crown, the other of flint glass, with the apex of one to the base of the other, since the ratio between the refractive and dispersive power is different in the two kinds of glass.* But though suggested long ago, such combinations have been precluded by their weight from clinical use.

* The dispersive power of crown glass is to that of flint glass as 33 to 52; whereas the refractive power is about as 31 to 32, though it varies greatly with different specimens, and must be determined for each.

Chapter III.

PRISMETRY

Apart from the question as to whether it is wise to *prescribe* prisms by their deviating angles, it is certainly of advantage for the surgeon to possess means of noting the optical strength of the prisms he uses. This is especially the case in scientific enquiries.

As the measurement of the physical angles of prisms has been known by the name of "goniometry," I have chosen the name of prismetry for the measurement of their optical power, to save confusion. The simplest form of goniometer is the "prism-measure" of the Geneva Optical Company.

But this acquaints us only with the geometrical —not with the optical properties of prisms; what the surgeon needs alone to know is not their refracting angle, but their deviating power over a beam of light. It has been noticed already that this "angle of deviation" is in each case about half the angle of refraction—with weak prisms inconsiderably more, and with strong ones increasingly more, than half,—but apart from this difference, the refractive index of different specimens of crown glass varies a little, so that Donders* truly observed

* "Accommodation and Refraction of the Eye" (Sydenham Society, p. 132).

of the angles of deviation, "If we wish to know these accurately, it is necessary to determine the deviations for each glass separately." To do this with certainty and ease I contrived two simple methods some years ago, one most suitable for practice, the other for demonstration purposes. The first can be made without expense, and employed without artificial light. It consists simply of a strip of paper or cardboard suspended horizontally on the wall, at such a height from the floor as to be about level with the eyes of the observer. The card itself is marked, as in *Fig.* 16, in tangents of degrees* for this distance. All that is necessary to measure the deviating power of a prism is to

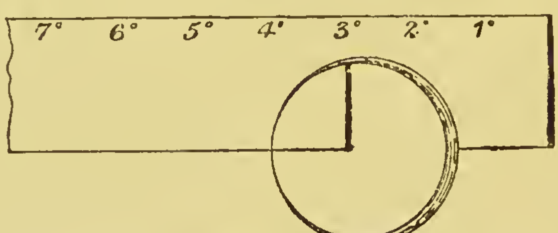

Fig. 16.—First mode of prismetry.

hold it at this distance† with its apex to the left, and so that its upper border appears just beneath the line of degrees, as in the figure. The vertical border of the card appears necessarily displaced

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* For tangents of degrees, *see* p. 161. A scale of this kind has been used since May, 1886, by Curry and Paxton, to whom I gave it, for measuring the deviating angles of their prisms.

† The distance of the observer's eye behind the prism does not affect the result in any way.

towards the edge, and points upwards to the number which expresses in degrees, at once and without calculation, the required deflecting angle of the prism. In the figure this is seen to be 3°. One precaution alone needs to be observed: if the lower edge of the cardboard seen through the prism appears disjointed from the level of the rest, as in *Fig.* 17, the apex of the prism is not pointing exactly to the left, but is either too high or too low, according as that part of the broken line seen through the prism is above or below the remainder. A slight rotation is therefore necessary to correct this, till the lower border of the cardboard appears at one level, as in *Fig.* 17.

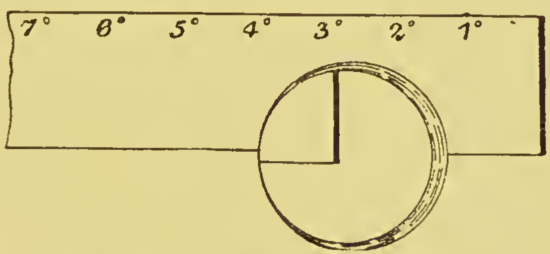

Fig. 17.—A prism not held truly.

The distances between the degrees of the card depend, of course, on the chosen distance at which it is found most convenient to hold the prism, and this would depend on the size of the room and other conditions. The distance of the observer's eye from the prism is of no consequence; anyone can easily mark a piece of cardboard for himself from the accompanying table which gives the

distances from the right-hand border of the card at which the several degrees are to be marked upon it, assuming the prism to be held at a uniform distance of six feet (first column), or two metres (second column).

TABLE II.

For marking a card in tangents of degrees at 6 ft. (column A); or 2 metres (column B).

	A	B		A	B
1°	1·25 in.	3·49 cm.	9°	11·4 in.	31·68 cm.
2°	2·5 „	6·98 „	10°	12·6 „	35·26 „
3°	3·7 „	10·48 „	11°	14·0 „	38·88 „
4°	5·0 „	13·98 „	12°	15·3 „	42·5 „
5°	6·3 „	17·5 „	13°	16·6 „	46·17 „
6°	7·57 „	21·0 „	14°	17·9 „	49·86 „
7°	8·84 „	24·56 „	15°	19·3 „	53·59 „
8°	10·12 „	28·11 „	16°	20·64 „	57·35 „

For a different range, the measurements would of course differ in proportion: thus, should it be decided always to hold the prism at 3 feet, the distances would be half as great as in the table; if at 12 feet, twice as great.

A second method of prismetry affords a much prettier *demonstration* of the deviating angle of prisms, but needs a more concentrated illumination than diffused daylight. It consists of a strip of wood, as in *Fig.* 18, laid flat on a table or counter with a shorter strip *b*, fixed at right angles to its further end in such wise that its plane is vertical. Upon the longer strip a cylindrical lens *c*, is fixed erect, with its axis vertical, so as to throw a line of

light from the gas flame *g* upon the strip *b*, which is marked in tangents of degrees; so that a prism held before the cylinder *c*, with its base-apex line horizontal, at once makes the bright line move to that degree which indicates the deviating angle.

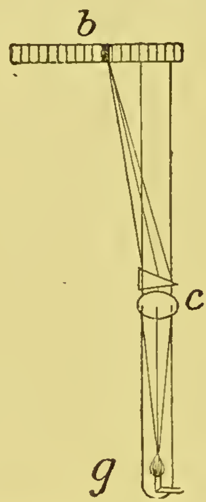

Fig. 18.—Second mode of prismetry.

The flame can either be fixed to the main strip or be quite separate, and it is well, if the light is poor, to combine with the cylindrical lens a spherical one of such strength that the source of light shall be at its principal focus.

Unit of Prismetry. Three proposals are in the field as to the unit of measurement for the deviation of prisms—the degree, the centrad, and the prism-dioptre. I leave the decision as to which possesses greatest merit to others, and will here only briefly notice some of the peculiarities of each.

The *degree* is a familiar unit, and if "possession is nine points of the law" should hold its own, unless some

really surpassing merit attaches to its rivals. We measure every other ophthalmic angle in degrees, and it would certainly seem strange to measure a deviation of the eyes in one unit, and correct that deviation in another. Perhaps when some other unit has established its superiority sufficiently to replace the degree for *all* purposes it will be time to accept it for prisms.

The second proposal is the "Centrad," by Dr. Dennett, of New York. It possesses three advantages over the degree.

1.—Being a new unit, and since it is only proposed to use it for the deviations of prisms and not their physical angles, it might lessen the possibility of confusing the two methods of marking prisms.

2.—It is a smaller unit than the degree, and a prism with a deviation of one centrad would answer roughly to a prism with a refracting angle of 1°. A centrad is slightly more than half a degree (·57 . . . of a degree).

3.—It has a slight relation to the metre-angle, in that half the number of centimetres between the pupils indicates the number of centrads in one metre angle.

The centrad is taken from the well-known "circular measure." It is the hundredth part of a "radian," a radian being the angle subtended at the centre of a circle by an arc which is equal in length to the radius. Circular measure has a certain theoretical interest.

The third unit, proposed by Mr. C. F. Prentice, is the "Prism-Dioptre." It has much to recommend it. The size of a single prism-dioptre differs inappreciably from that of a centrad, both being the departure of 1 in 100, that is, a deviation of one centimetre at one metre distance, but the centrad is measured on the arc, and the prism-dioptre on the tangent. The relation to the metre-

angle of the P.D. is practically the same as the centrad, though not quite so perfect.* Its chief recommendation is the ease of measuring on the straight, and also its beautiful relation to lenses, and their decentering. It has the disadvantage, however, of not being theoretically suitable in high numbers for multiplication: for instance, two prism-dioptres form an angle less than twice times one prism-dioptre. A prism-dioptre is 34′ 35″.

The *measurement* of prisms is as easy for one unit as for another, and the scale recommended on p. 30 has been adapted by Prentice to the prism-dioptre, by having it marked in centimetres instead of in degrees, while a beautiful instrument has been constructed by him called the "Prismometer," to use where great refinement is wanted. To measure in centrads, a short arc would have to be used, marked in hundredths of its radius, or a tangent scale could be made in tangents of centrads.

The adoption of the prism-dioptre would certainly reduce the decentering of lenses to extreme simplicity, and this is by far the strongest point in its favour.

A lens of 1D displaced laterally one centimetre has the effect of a 1 P.D. prism; displaced 2 cm. of 2 P.D., and so on. A lens of 2D similarly displaced, has the effect of a 2 P.D. prism.

We should have only to remember that the prismatic effect is that of the decentering in centimetres, multiplied by the dioptric strength of the lens.

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*The metre angle is the angle whose *sine* is half the inter-ocular distance in terms of metre. Since the centrad is an arc measurement, and the prism dioptre is a tangent measurement, the former approaches the sine more nearly than the latter. Arcs lie between sines and tangents. (See Appendix, p. 159.)

CHAPTER IV.

TESTING THE APEX.

CIRCULAR prisms, as usually sold, have their apex and base indicated by a slight scratch from a diamond. On testing a series of prisms by a simple plan, to be described immediately, much inaccuracy was noted in a large proportion, and experimental work may be sometimes vitiated on this account. An obvious consequence of inaccurate marking, is that prisms may not be quite squarely set in the trial-frame; yet the slightest departure from precision in this particular alters an experiment. For example, if it be desired to see

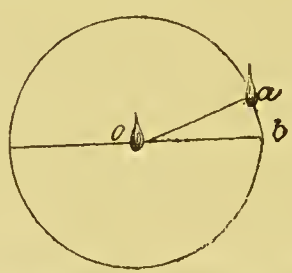

Fig. 19.—Shewing the effect of setting a prism by a wrong mark. The marked apex is towards *b*, and the true apex is towards *a*, so that the flame *o*, which should appear at *b*, appears at *a*.

whether a patient can overcome a prism of 15° d, with its apex exactly in or out, an aberration of the base-apex line through only a ninetieth part

of a circle has the effect of not only lessening in a trifling degree the desired horizontal deflection which it is the aim of the prism to produce, but also of introducing a new and unsuspected vertical deflection of more than 1°. This is shewn in *Fig.* 19, where the actual flame *o* appears displaced to *a* instead of *b*;—the true base-apex line being *o a*, though the prism is marked as if it were *o b*; hence *a o b* is the angle of error in setting the prism. At twelve feet distant the flame would appear more than two and a half inches higher than it really is.* Now it is well known that vertical diplopia greatly embarrasses the neuromotor apparatus, and in some persons it cannot be overcome at all when greater than one degree, so the prism in the experiment may be put down with the verdict that it is too strong to be overcome, when really it is the concomitant accidental *vertical* diplopia created by it which cannot be overcome, or at least the effect of which alters the result.

If anyone will take the trouble to test the marks upon his own prisms, he will find how frequently

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* The vitiating vertical deflection of a ray of light by a prism whose base-apex line is supposed to be horizontal, or the vitiating horizontal deflection by a prism whose base-apex line is supposed to be vertical, may be found in any particular case, by the following formula, $\sin. x = \sin. D + \sin. r$; where x is the angle required, D is the deviating angle of the prism, and r the angle of accidental rotation from exact rectitude.

they are misplaced ; the test can be easily made by the following simple device. Draw upon a sheet of paper two fine straight lines intersecting each other exactly at right angles, as in *Fig.* 20, and hold the prism horizontally above them, so that on closing one eye the supposed apex apparently coincides with the point of intersection of the lines, and the prism appears to touch the horizontal line only at that point, and does not in the least overlap it on either side. If the prism is correctly marked, the vertical line appears unbroken, as in *Fig.* 20, but if not, the part of the line seen through the prism appears

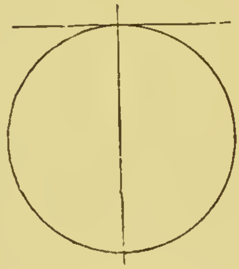

Fig. 20.—Mode described for testing the apex of a prism.

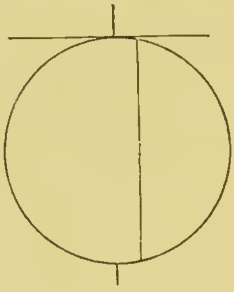

Fig 21.—Appearance with a badly marked prism.

disjointed from the rest and displaced, as in *Fig.* 21, towards the side of the real apex. It only remains to rotate the prism till the unbroken appearance of *Fig.* 20 is gained ; then the point on the prism's edge in apparent coincidence with the point of intersection of the lines may be regarded with confidence as the real apex.

Greater distance of the prism from the vertical

line magnifies the phenomenon, and makes the test more delicate. The true base of a prism can be found and marked in just the same way as the apex, and if prisms thus manufactured come into use, one of the many causes of experimental inaccuracy will be removed. To those who manufacture prisms in large quantities it may be permissible to suggest a special apparatus to mark the apex with mathematical accuracy. Each prism should be laid on a horizontal glass plate, with cross lines beneath, or, better still, one longitudinal line, the use of the transverse line being replaced by two pins in the glass plate, half an inch apart, to press the prism against, while it is rotated till the longitudinal line appears continuous, then a diamond held in a sliding arm, and working midway between the two pins, would mark the apex.

Chapter V.

PRISMATIC ABERRATION.

With the exception of chromatic aberration (p. 27), which prisms share in common with lenses, the optical imperfections of prisms are chiefly due to the fact that no constant ratio exists between the angles of incidence and refraction, but only between the sines of those angles (p. 162). We have seen on page 17 that parallel rays, entering a prism, also emerge parallel ; but with this exception, all homocentric light, that is, all light proceeding from, or going towards, a single point, is, after traversing a prism, no longer homocentric. The stronger the prism, and the greater the departure of the beam of light from the direction of minimum deviation, and the more converging or diverging its component rays, the greater is the prismatic astigmatism. With weak prisms, as used in practice, it gives rise to little more inconvenience than the spherical aberration of lenses.

But it accounts for some phenomena, which are apt to puzzle those who experiment with strong prisms. The astigmatism of a prism is slightly akin to that produced by a weak cylindrical lens, in that a cone of light, after having passed through

a prism, is somewhat flattened, its section becoming slightly elliptical instead of circular; or, in other words, two rays which enter the eye in one meridian have a more distant focus than two which enter it in another meridian. In *Fig.* 22, two rays proceeding from a point *o* enter the pupil of an eye in the

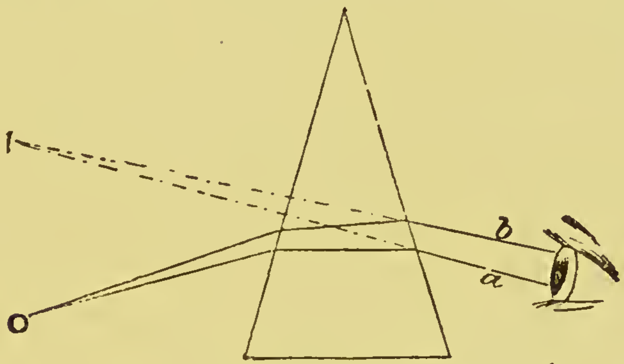

Fig. 22.

vertical meridian. The inferior ray (*a*) is drawn so as to traverse the prism in the direction of minimum deviation, and it therefore experiences the least possible deflection with which any ray can pass through a prism.

Since the superior ray (*b*) has a different angle of incidence, it experiences a greater deflection, so that on their emergence the two rays are less divergent than they were before. On this account they enter the eye as if they came from a point (*I*) a little further off than it would be if there were no prismatic astigmatism.

Suppose now the prism were tilted a little round

its axis, so as to bring the superior ray, instead of the inferior, into the direction of minimum deviation. The conditions would then be reversed, for the inferior ray would be most deflected, and the two rays would enter the pupil more divergent, and as if they came from a point *nearer* than they would do if there were no prismatic astigmatism. We see, therefore, that the nature of the astigmatism is altered according as the apex of the prism is moved (away from the position of minimum deviation) towards the object, or away from the object. The greater, too, the tilting, the greater the astigmatism.*

One consequence of prismatic astigmatism is that objects appear altered in shape when viewed through a prism. If square objects are looked at through a tilted prism, they appear oblong, being narrowed in one dimension, if the base of the prism be anterior

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* What about the rays which enter the horizontal meridian of the pupil? Even they are not simple in their behaviour, but the more peripheral ones traverse the prism higher (*i.e.*, nearer the edge) than the central ones, so that they describe an increasingly curved surface as they proceed from *O* till they reach the prism, and after leaving it, this transverse curve (with its concavity upwards) becomes less and less, till they enter the pupil in the straight line of its horizontal diameter.

This will be easily understood after the explanation of the second form of prismatic aberration, since it depends on the fact that the peripheral rays have, after leaving the prism, a longer course than the central ones.

to the apex, and broadened if the apex be much anterior to the base. If a scale of equi-distant figures on the wall be viewed through a prism with its apex to the right, the intervals between the right hand figures will appear larger, and those between the left hand figures smaller, than they should be.*

An undescribed form of prismatic aberration is observed when a long straight line is looked at through a prism. It appears curved, with its concavity towards the apex of the prism. Let us endeavour to give an explanation of this phenomenon, and in language which will make it intelligible to an unmathematical reader. Take a paper triangle, as in *Fig.* 23, and draw a line bisecting its acutest angle, such as *a f.* Then, if we imagine the base (*b c*) of the triangle to be the long

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* It will be easy to understand this from *Fig.* 22, if the eye there represented be imagined as the object looked at by an observer's eye at *O.* If the prism be in a position, such that the most apical ray (*b*) is deflected more than the basal ray (*a*), objects will appear vertically lengthened, while, if the basal ray were most deflected, objects would appear vertically shortened. In this use of the figure, the rays of light represent "lines of direction," meeting at the nodal point of the eye, and including between them the "visual angle." The apparent size of objects depends on the size of the visual angle, and it is evident that this would be larger when the apical "line of direction" is bent most, and smaller when the basal one is bent most. Each line of direction is of course one ray of a cone of light, the base of which is the observer's pupil, and the apex some point of the object.

horizontal line we are looking at, and *a* the position of the observer's eye, the lines *a b*, *a f*, and *a c* represent three rays of light as they would reach the eye were no prism in the way. We may, for our purpose, suppose the size of the pupil to be inconsiderable, so that rays from every point of the

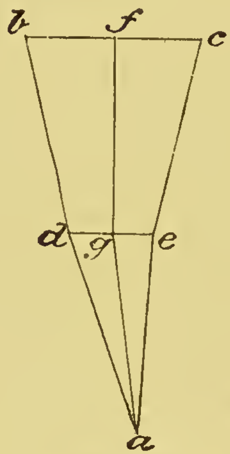

Fig. 23.

line travel to the eye at *a* in one plane—the plane of the paper. Now bend the triangle at *d e* over the sharp edge of a table, and we have what at first sight we might think to be the alteration in the direction of the light to be introduced by interposing a prism, edge up, in the position *d e*; for the prism deflects every ray of light towards its base, one as much as another.

The paper triangle does not, however, truly represent the case, for if it did we should not see the line curved but straight, since the light would still

enter the eye in one plane, for the smaller triangle *a d e* is just as much a plane as the whole triangle was at first. What, then, is the explanation? It is this, that the distances *a d* and *a e* being greater than the distance *a g*, the *gradient* is less, as even a horse knows who prefers to descend a hill obliquely. A fly walking down *e a* would not have so steep a walk as one walking down *g a*. The paper, therefore, does *not* represent the course of the rays through a prism, for a prism imparts equal gradients (that is, equal steepness) to all the rays. For the rays *d a* and *e a* therefore to possess the same gradient as *g a*, they must traverse the prism higher up, just as a long hill is higher than a short one of equal steepness. If we took a triangle of elastic material, and stiffened the base of the triangle, and then bent it at *d e* over a *curved edge*, with its concavity upwards, we would then represent the course of the rays of light, and show why the line appears curved.

So far we have only spoken of prisms held before one eye, the other being closed. *Binocular distortion* results when both eyes are fitted with prisms, and has received its partial explanation, I believe, from Wadsworth, to whose paper* I would refer those interested, since it is difficult to give a just account of it without reproducing

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* Trans. Amer. Oph. Soc., 1883.

his diagrams in full. It depends on the different deviations produced with different angles of incidence, as already described in speaking of prismatic astigmatism, and accounts for the spherical appearance of flat surfaces when they are viewed through prisms.

Since tangents increase faster than their angles (p. 159), it follows that when we look with one eye at a plane surface, the details of that surface are more crowded together in the peripheral portions of the retinal image than in the central portions. But we have learned to disregard this, and consider it normal. Prisms with edges *out* disarrange this peripheral overcrowding, in binocular vision, in a way which reminds us of a spherical or cylindrical surface. Not only so, but the eyes in glancing from detail to detail, make different excursions when viewing the periphery of a spherical surface, than that of a plane surface, in proportion as the tangent to the surface is inclined to the visual axis. A similar alteration in the excursions, when we turn the eyes from side to side, takes place in consequence of wearing prisms. It is really the altering of the motions of the eyes which accounts most, I believe, for the illusion. It is most marked when prisms are worn edge out; a plane surface then appears convex towards the observer. With prisms edge in a plane surface appears concave.

CHAPTER VI.

ROTATING PRISMS.

WE are indebted to Sir John Herschel for showing how, by placing two prisms in apposition, and rotating them in opposite directions, we can produce the effect of a single increasing prism. If two prisms of equal strength be placed with the apex of each against the base of the other, they so neutralise each other as to have the effect of only a thick plate of glass. The more they are rotated from this position, the greater is the prismatic effect of the combination, till it reaches its height, when the two apices coincide. Crétès, of Paris, mounted two such prisms in a circular frame with a handle, so that on moving a button along the handle the prisms revolved in opposite directions at equal rates. This useful instrument is commonly known as "Crétè's prism," or the "prisme mobile." Each prism has a refracting angle of 12°, and the instrument is marked accordingly. Landolt has had it marked in degrees of deviation, and metre angles. If we wish to know how strong a prism can be placed before one eye without creating diplopia, we start at zero, and press up the button till double

vision commences, when the strength of the prism can be read off.

By far the neatest form, however, in which Herschel's prisms are put up, is that of the "Rotary prism," designed by Dr. S. R. Risley, of Phila-

Fig. 24.—Risley's Rotary Prism.

delphia. It is small and circular, and can be placed in an ordinary trial-frame. On turning the little button the two prisms, each of 15°, rotate in opposite directions. The diameter in which the button lies coincides with the base-apex line of the *virtual* prism, however much the actual prisms are rotated. The virtual prism can be increased from 0° to 30° (15°d). The markings indicate the physical angle of the virtual prism, not the deviating angle. If the apex of the front actual prism be to the button side of zero, the position of the button indicates the apex of the virtual prism; if the actual apex be to the other side of zero, the button indicates the base

of the virtual prism. Every form of Herschel's prism, however, has some disadvantages. The diminution in the visual acuity, occasioned by the chromatic aberration and prismatic astigmatism, is in one eye only, and thus the desire for single vision is lessened more than if it were equally distributed over both eyes, as in the apparatus sketched in *Fig.* 25, where one prism revolves in front of each eye. This apparatus would only be of use for testing convergence and divergence. To use it, we would start with both prisms, apex upwards, and rotate the apices towards, or away from, each other. The plane in which both visual axes lie is depressed in so doing; but this is of no account, while in addition we obtain the increased convergence or divergence of the visual axes which would

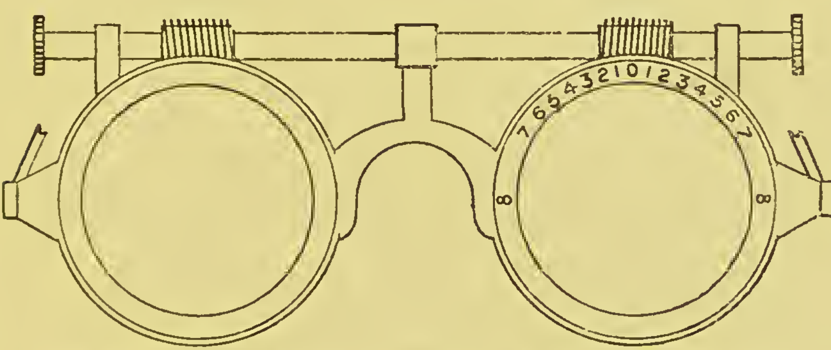

Fig. 25.—Proposed rotary prismatic trial frame with prisms before both eyes.

be produced by prisms with their apices inwards or outwards. In the following table I have shewn how much each prism needs to be rotated to give

the horizontal effect of 1°d, 2°d, etc., supposing the strength of each actual prism to be 8°d.

TABLE III.

Rotation Required.	Horizontal Effect.	Rotation Required.	Horizontal Effect.
7° 8′	1°d	38° 30′	5°d
14° 23′	2°d	48° 24′	6°d
21° 53′	3°d	60° 53′	7°d
29° 50′	4°d	90°	8°d

The table is made from the formula below, in which *H* is the required horizontal effect for *each* prism, *D* the deviating angle of each prism, and *r* the rotation of the axis of each from the vertical. Then

$$\sin. r = \frac{\tan. \text{H.}}{\tan. \text{D.}}$$

For sines and tangents, the reader is referred to page 161. Since the second column in the table gives the deviation imparted to *each* line of fixation, it must be multiplied by two to give the total effect on convergence, and to compare it with Crétè's or Risley's prisms it must again be multiplied by two, since these are marked according to the refracting angles and not according to the angles of deviation.

CHAPTER VII.

IMAGES BY INTERNAL REFLECTION.

HITHERTO we have only considered the *refraction* of light by prisms, but at each surface a smaller portion is also reflected. The more oblique the incidence of a beam of light, the greater is the proportion reflected, and the brighter is the image produced by reflection.

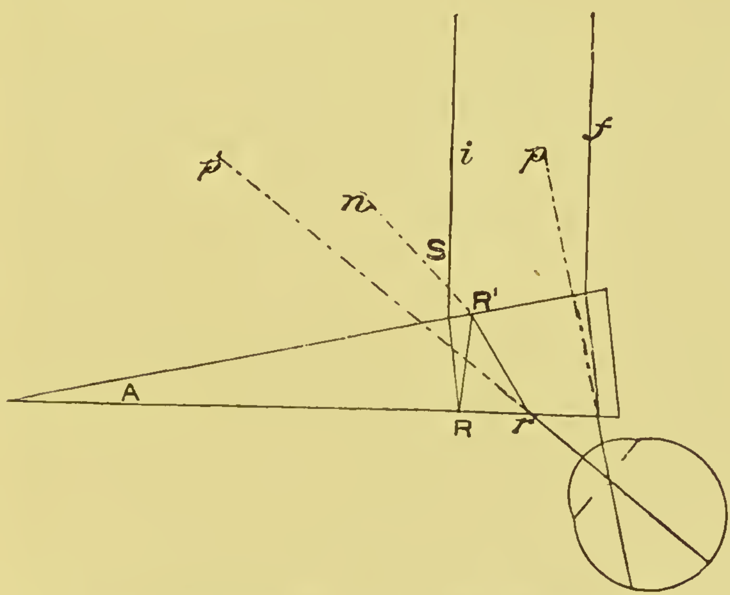

Fig. 26.—Image produced by internal reflection within a prism.

The light reflected from the first surface of a prism is lost to the eye altogether, being returned

into space, but that which, after refraction at the first surface, undergoes internal reflection at the second surface, as at *R* in *Fig.* 26, is again reflected internally from the first surface at *R'* (though at each surface some escapes by refraction) before it finally reaches the eye by refraction at *r*. Even at *r* a portion is again reflected internally, and this internal reflection from surface to surface goes on till the base of the prism is reached, or till all the light is, for practical purposes, lost by escaping from the prism. With a weak prism we may, on looking through it at a flame, see a series of images from internal reflection, especially if we hold it obliquely to increase their brightness, but each becomes fainter than the last. They, and the flame, all appear in one straight line, which is parallel with the base-apex line of the prism. Only the first of these images is illustrated in *Fig.* 26.

When the incident ray *(i)* traverses the prism in the direction of minimum deviation, the angle of reflection at *R* is equal to the refracting angle *(A)* of the prism, and each subsequent reflection is by an angle greater than the last by twice the angle of the prism (3A, 5A, 7A, etc.). It is under these conditions that the course of the rays can be studied to most advantage, for one finds by experiment that moderately tilting a prism away from this position has very little effect on the apparent position of the image.

Since the ray which escapes at *r* into the eye

determines the direction in which the faint image is projected, the projection is indicated by the dotted line p'. The ray f is one which enters the eye by refraction only, and is projected in the direction of the dotted line p. The angle between the two dotted lines p and p' represents, therefore, the apparent visual angle between the bright image of the flame seen by refraction only, and the first faint image seen by reflection as well. Since the imaginary line joining these two images is exactly parallel to the base-apex line, we are provided with a very beautiful way of setting a prism exactly horizontal or vertical: we have only to see that these two images are horizontal or vertical relatively to each other. In other words, the faint image of a flame viewed through the centre of a prism points exactly to the apex of the prism, and it is curious that so simple a guide does not appear to have been hitherto utilized. Could we not utilize also the *distance* between the bright image of a flame and its faint image for measuring the strength of a prism? For the stronger the prism the greater their distance. Nothing is easier than to say what figure of a scale is covered by the faint image of a flame placed at the zero of the scale and looked at through a prism, and were the refractive index of glass a constant quantity, this plan would provide us with a more delicate test of the strength of a prism than any in use. But unfortunately

there are slight differences in the refractive index, and the visual angle depends not only on the angles of reflection, but also on those of refraction. The formula I have made, however, shows that the figure on a tangent scale pointed out by the faint image, indicates, without much error, that the angle of the prism is nearly one-third, and the deviating angle nearly one-sixth, of that expressed by the scale. If we know the index of refraction, or have such confidence in the uniformity of the glass as to take its average for granted, we can find the exact strength of the prism by a formula. The distance of the scale should not be less than a metre, preferably two or five, if great delicacy be required.

I will endeavour to show how I constructed a formula. Let a line be drawn from R', such that if it represented a ray incident at R' its refracted ray would be $R'r$. Then the angle *(S)* between this line and the ray i, is the angle expressed by the figure on the scale covered by the faint image, for it is evident that light proceeding from n, and light proceeding from i enter the eye as one in the beam from r. But the ray $R'r$ forms an angle of $\frac{3}{2} A$ with the normal, therefore the angle of incidence of the ray n is $sin.^{-1} (\mu \; sin. \frac{3}{2} A)$. This gives us the inclination of n to the normal. The inclination of i to the normal is evidently $sin.^{-1} (\mu \; sin. \frac{A}{2})$; but these two angles together compose S. So that $S = sin.^{-1} (\mu \; sin. \frac{3}{2} A) + sin.^{-1} (\mu \; sin. \frac{A}{2})$. From this a table could be easily constructed for any given index of refraction.

In the table which follows, I have taken the index of refraction as 1·54.

TABLE IV.

Refracting Angle of Prism.	Projection of Faint Image on a scale.
1°	3° 4′
2°	6° 9′
3°	9° 14′
4°	12° 21′
5°	15° 27′
6°	18° 34′

CHAPTER VIII.

CLINICAL PROPERTIES OF PRISMS.

THE only clinical use of prisms is to change the direction of the rays of light, but this is fruitful in its secondary applications. Prisms always produce an optical illusion, for they make objects appear in a different position from that which they actually occupy.

Clinical prisms, as we have seen, are generally circular, their thinnest point being called the

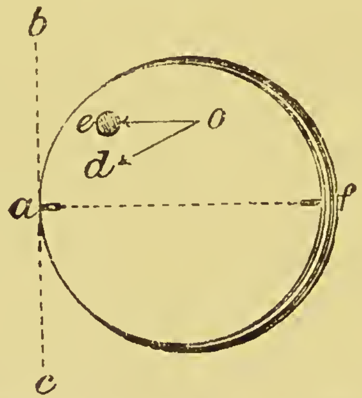

Fig. 27.—To shew how an object at (*O*) appears displaced, not in the direction *d* towards the apex (*a*), but in the direction *e* towards the edge (*b c*), and parallel to the "base-apex line" (*a f*).

"apex," their thickest part the "base," while an imaginary line between the apex and base may be called the "*base-apex line.*" All objects viewed

through a prism appear displaced in a direction parallel to this line and always towards the edge. It is customary to say that objects appear displaced towards the *apex*, and in practice there is no harm in using this convenient though less correct mode of expression, provided it be not misunderstood. *Fig.* 27 illustrates the difference, and shews that the object *o* does not appear displaced in the direction *d* towards the apex *a*, but in the direction *e* towards the edge *b c*, for while *o* represents the real position of the object, *e* represents its apparent position as seen through the prism. If the prism be rotated, its edge, of course, rotates with it, but in all positions the object will appear displaced to the same extent towards the edge, so that, in making a complete revolution of a prism, the false image will appear to revolve at an equal rate round the real position of the object.

Lines viewed through a prism.—If, instead of using a small object, we look at a vertical line on the wall, or the edge of a door, as *x x* in *Fig.* 28, through a prism held several inches from the eye with its apex at *a*, we observe that the part of the line seen through the prism appears disjointed from the rest in a direction towards the edge of the prism, but the disjointed portion still appears a nearly straight line. If another line, such as *y y*, be in view through the prism at the same time, its disjointed portion will appear displaced in the same

direction, and to exactly the same amount as that of the other line $x\,x$. It will be noticed too, that movement of the prism from side to side will not affect, in the slightest degree, the apparent position

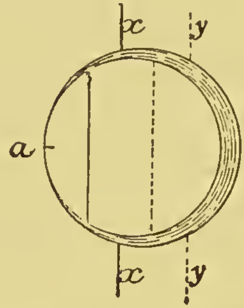

Fig. 28.—To show the apparent displacement of lines through a prism.

of the disjointed portions of the lines. Herein a prism widely differs from a lens, for since the refracting surfaces of any prism are of necessity inclined to each other by the same angle in every part of their area, it follows that the apparent displacement of an object is the same through whatever part of the prism it is viewed. It is true that rotation of the prism will alter the apparent position of the disjointed portions of the lines, but they will still continue (if the prism be weak) vertical. Herein again a prism differs from a spherical lens, rotation of which has no apparent effect whatever, while a cylindrical lens differs also from a prism, in that when it is rotated, a line viewed through it appears distorted from the vertical. When a prism

is rotated, the apparent displacement of the disjointed portion varies in amount according to the rotation. The maximum displacement is obtained when the base-apex line is at right angles to the line looked at, and the displacement becomes less and less as this position is rotated from till the base-apex line becomes parallel with the line viewed, in which position the displacement is *nil*, or, rather, it appears *nil*, for, in fact, there is just as much displacement, but being in the direction of the whole line, it is not evident.*

ANGLE OF FALSE PROJECTION.—Since the apparent position of an object in space is determined

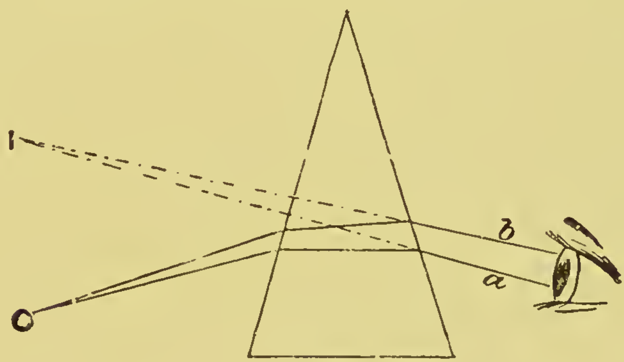

Fig. 29.

by the direction in which the rays of light from the object enter the pupil, it follows that, when viewed

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* It will be noticed that a long straight line, viewed through a prism, held about an inch from the eye, with its base-apex line at right angles to it, appears curved, with its concavity towards the apex. This is explained on *page* 43.

through a prism, the object will appear misplaced by an angle which is nearly identical with the deviating angle of the prism.

In *Fig.* 29 the real position of the object is at *O*, but it appears to be at *I*, because the pencils of rays enter the pupil as though they came from *I*.

The angle by which an object appears displaced ("angle of false projection") is always less than the actual deflection of light by the prism, owing to the distance between the prism and the centre of rotation of eye, as shown in *Fig.* 30, where the deviating angle of

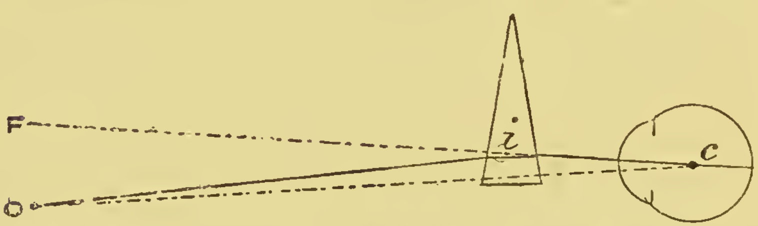


Fig. 30.—To illustrate the discrepancy between the deviating angle of a prism and the apparent displacement of an object seen through it.

the prism (*o i F*) is evidently greater than the angle of false projection (*o c F*). But generally the distance of the object is so great compared to the distance of the prism, that the two angles may be taken as practically identical.

Displacement of the visual axis by prisms.—We have seen that prisms appear to *displace objects in the direction of their edge.* Since an eye tends to be diverted towards the (apparently) new position of the object, it follows that *the line of fixation is also displaced towards the edge*, and in working with

prisms these two clinical properties are all that need be remembered. *Fig.* 31 illustrates both : a

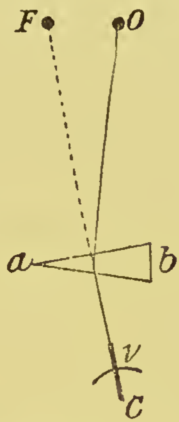
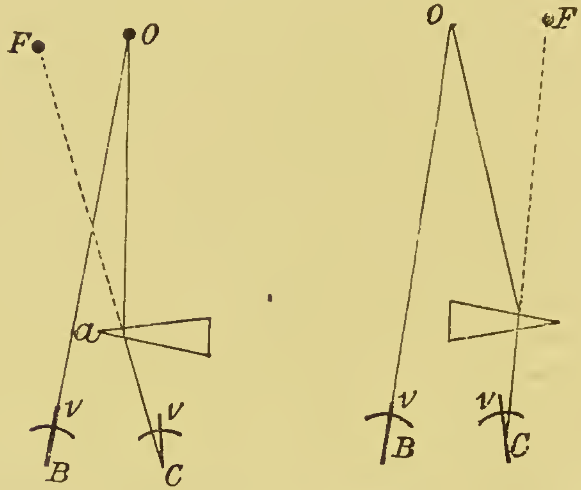


Fig. 31.—A prism before an eye, in monocular vision.

prism is placed before one eye, *the other being covered*, and it is evident that the object *O* appears displaced to *F*, and the line of fixation *v* is diverted from *O* to *F*,—both in the direction of the apex *a*. The line *F C* in which the false image lies is called the "line of projection," since it marks the direction in which the picture on the retina is mentally projected.

Overcoming of Prisms.—With *binocular* vision, as in *Figs.* 32 to 35, the conditions are not quite so simple. In all these figures the continuous lines shew the actual course of the rays from the object *O*, and the dotted lines are the lines of projection. They almost explain themselves : in all it will be seen that the object appears displaced towards the edge of the prism, though in some (*Figs.* 32 and 35)

there is diplopia, since the object is seen in its true position *O* by the naked eye at the same time that its false image *F* is seen by the other in the

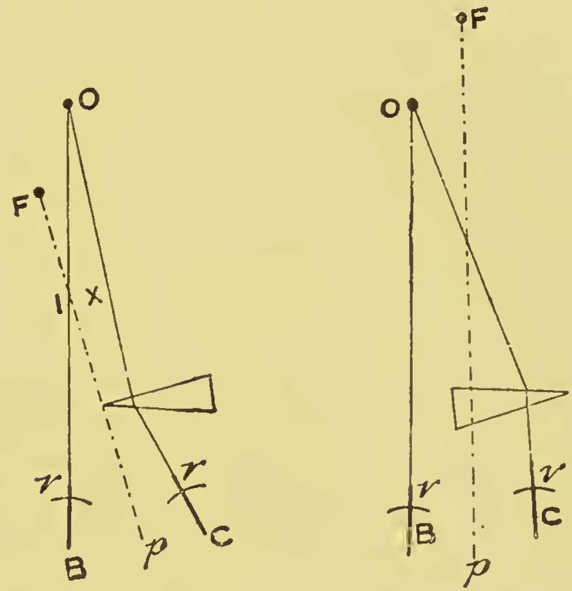


Figs. 32 and 33.—Prisms not overcome, and therefore causing diplopia.

direction of the edge of the prism, which has its edge *in* in *Fig.* 32, and its edge *out* in *Fig.* 33. In these two figures the conditions are those in which a prism is not "overcome," that is, the desire to unite the true and the false image into one is not great enough to cause the visual axis (*v*) of the right eye (*c*) to accommodate itself to the new direction of the pencils of light from the object; or, to put it another way, the association between accommodation and convergence is too strong for the eyes to accommodate for *O*, and yet at the same time, converge for a much nearer point (*Fig.* 32) or a much farther point (*Fig.* 33). There

is, as a necessary consequence, double vision. The eyes have, as it were, to choose between double vision on the one hand, and the maintenance of an abnormal relation between convergence and accommodation on the other. With weak prisms they prefer the latter, but with prisms above a certain strength they have to put up with the former.

In *Figs.* 34 and 35 the prisms are "overcome,"



Figs. 34 and 35.—Prisms overcome; no diplopia.

and there is no diplopia; it has been corrected by the desire for single vision, and the two images are fused into one. Now, the *united* image appears in a false position, though the angular displacement of the line of projection, *p F*, is only half that of the false image alone in the previous examples. The diplopia is corrected by a rotation of the eye so as

to receive again upon its fovea centralis the picture of the object which the prism had dislodged to another part of the retina; thus in these two figures the line of fixation, *v C*, has come to be in line with the rays of light coming through the prism from the object, and since both eyes receive a picture on the fovea, vision is single. In *Figs.* 32 and 33 there is *no* definite line of fixation for the right eye, since it is fixing nothing, and it is clear that the picture must fall on the retina away from the fovea; though with every effort to unite the images and bring the axis of vision, *v C*, into the line of the rays, the position of the picture on the retina will vary, and the images will move towards or from each other.

Prisms set with their edge inwards, as in *Figs.* 32 and 34, are called *adducting prisms*, and those with their edge outwards, as in *Figs.* 33 and 35, are called *abducting prisms.* The former, when vision is binocular and without diplopia, by increasing the necessary convergence of the eyes, make objects appear nearer than they actually are, and the latter make them appear too far away. Thus in *Fig.* 34 the real object *O*, appears to be at *F*.* As Hering has shewn, convergence occurs as if for the cross, and both eyes are turned towards *I* by an

* Theoretically the combined image should be at *I*, were convergence the only criterion, but accommodation remains for a greater distance, and the "knowledge of distance" removes its apparent position from *I* to *F*.

impulse of the conjugate innervation which turns the two eyes to the left: it is the mental cognizance of *this* effort which imparts corresponding obliquity to the line of projection, *p F*. The prism, therefore, does not test the strength of either rectus, since convergence is a single action affecting both eyes equally, but it estimates by how much convergence can be made to exceed accommodation, or *vice versa*. The two functions are so associated in daily work that there are limits to the amount by which either can be increased or diminished without the other. Prisms should be placed as close as possible to the eye, then their angles of deviation nearly express the diplopia, or corrective squint, they occasion, or the sum of both if both are present.

One of three events, therefore, may happen on placing a prism before an eye. (1,) If vision is monocular, one false image is seen, whose distance from the actual position of the object is unvarying, and dependent entirely on the deviating angle of the prism, *Fig.* 31. (2,) With binocular vision the prism may be too strong to be overcome, in which case there is diplopia with an inconstant distance between the images, which approach each other spasmodically with every effort to overcome the prism. (3,) The diplopia may be overcome and the object appear single, though its position in space is misjudged through an angle practically equivalent to half the deviating angle of the prism.

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Chapter IX.

ADJUSTMENT OF PRISMS IN THE TRIAL-FRAME.

If the apex and base of a circular prism are correctly marked, there is no difficulty in setting it with the base-apex line either exactly vertical or horizontal. But if there should be any doubt about their precise position, the following simple expedient will at once enable us to detect and correct any malposition. If the base-apex line is wished to be horizontal, as with the right hand prism of *Fig.* 36, hold the trial-frame a few inches from the

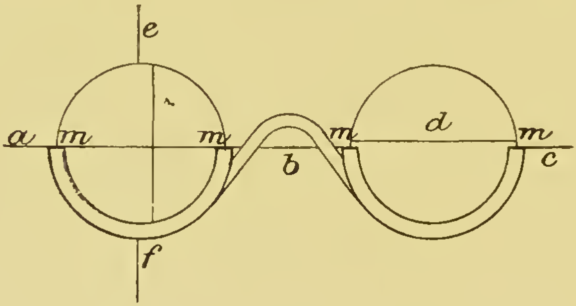

Fig. 36.—Two prisms in a trial-frame.

eyes, so that, as represented in the figure, the upper extremities *m m m m*, of its sockets coincide to all appearance with a horizontal line *a b c*, upon the wall, door, or window: if this line appears con-

tinuous, the prism is correctly set, but if that part of it visible through the prism appears disjointed from the rest, as *d* in Fig. 36, the apex is either too high or too low, according as the apparent disjointment is upwards or downwards. In the figure the apex is too high, and the correction is therefore easily made by rotating the prism so as to depress the apex till the line appears continuous. If the base-apex line is wished to be vertical, as with the left hand prism of the same figure, the horizontal line on the wall must be supplemented by another at right angles to it, as *e f* in the figure: again adjusting the frame as before to the horizontal line of sight, the rectitude of the prism is now indicated by the appearance of the *vertical* line. If its course appears undeviating, the setting is faultless, but if, as in the figure, the part seen through the prism is dissociated from the rest, the apex is shewn to be too much to the same side, and it therefore needs readjustment. With artificial light, the images by double internal reflection can be utilized to great advantage, as described on page 53. The best way is to place the trial-frame on one's own face, and rotate the prism till the faint image appears level with the flame, or vertically above it.

RESULTANT PRISMS.—We sometimes wish to combine the effect of a vertical prism of known strength with the effect of a horizontal prism of known strength. By vertical and horizontal prisms

are meant prisms whose base-apex lines are respectively vertical and horizontal. It is true that we may place the vertical prism before one eye, and the horizontal one before the other. But as a rule it is better to divide each prism between the two eyes to equalise the weight. In that case, let R stand for the deviating angle of the resultant prism before each eye, and V and H for those of the vertical and horizontal prisms respectively for each eye; and let r be the angle from the vertical at which the base-apex line of the resultant prism must stand. Then

$$\text{Tan. } R = \sqrt{\text{Tan.}^2 V + \text{Tan.}^2 H}.$$

and $$\text{Sin. } r = \frac{\text{Tan. } H}{\text{Tan. } R}.$$

As Dr. Duane truly says, for the weak prisms we prescribe, the sine, arc, and tangent may be taken as identical without sensible error. Then the simpler formulæ suffice,

$$R = \sqrt{V^2 + H^2}$$

and $$\text{Sin. } r = \frac{H}{R}.$$

I am indebted to Dr. Duane for correcting a slight error which crept into this section in the first edition. For the simplest proceedings of all, see Appendix, page 158.

CLINICAL STRENGTH OF PRISMS.

The optical strength of a prism is invariable, but its clinical strength, or physiological strength, depends on the use that is made of it. For instance, when a pair of adducting prisms are worn in the usual way, their effect on convergence is least on looking straight forward to a distance, and becomes greater on looking to either side, and also on near vision. This is due to principles already explained (p. 24). When abducting prisms are worn, the effect of the pair on convergence is also greater on looking to either side than on looking straight forward, and there is a certain distance, dependent on their strength, beyond and within which their effect increases. These really unimportant variations may be lessened by placing the bases of adducting prisms slightly in advance of their apices, and the bases of abducting prisms slightly in advance for distant and in arrear for near vision.

There is, however, one difference between the optical and clinical strength of prisms, which is of some slight importance, and may perhaps be commended to those who prescribe prisms for hyperphoria.

A vertical prism, which fully corrects a hyperphoria in distant vision, only partially corrects it in near vision, for the clinical effect of a vertical prism, on the separation of the visual axes, is greater in distant than in near vision. This is shewn in *Fig.* 37, where the line *O N* represents the visual axis of

the naked eye, and the lines n and n', that of the other eye, on looking, through a prism, at a distant object, and a near object, in turn.

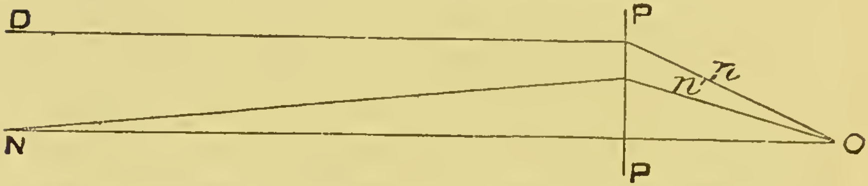

Fig. 37.—To shew that the clinical strength of a vertical prism is greater in near than in distant vision.

For practical purposes, since the position of the prism, and the direction of vision, are so inconstant, it will be better to make a simple approximate rule than to give useless formulæ, and the following one will be found quite sufficient if any be needed.

Divide the deviating angle of the prism by the distance in inches of the object from the centre of the eye to find the reduction in the effect of the prism in near vision. Thus, with vision for five inches, the elevation of the visual axis by a prism is one-fifth less than in distant vision, so that a prism of $5^\circ d$ would have an effect of 4° on the visual axis. With vision for ten inches, the elevation of the visual axis would be one-tenth less, and so on. It is well to know the fact thus treated of, because, if a patient fitted with vertical prisms says they suit him best in near vision, it may mean they are too strong, whereas, if they suit him as well in distant vision, they are probably not.

Chapter X.

DECENTERING OF LENSES;

AND COMBINATIONS OF PRISMS WITH LENSES.

Instead of imparting sphericity to the surfaces of a prism, and thus combining a prism and a lens, where it is wished to obtain the effect of both, it is possible to get the same result more accurately by simply decentering the lens, a process which, though already well known, may be simply explained.

The "optical centre" of a lens is that point traversed by all rays which, after entering the lens, emerge from it in a direction parallel to their former course.* All rays which do *not* traverse the optical centre are bent from their former course to a degree which depends on the distance from this point, at which they pass through the lens—the greater the distance the greater the deflection. In other words, a lens, in every part other than the optical centre, has the effect of a prism upon any

* This one "property" is all that is necessary for our present purpose, hence it is mentioned, instead of giving a correct definition. It is that point in the principal axis of a lens, which divides the line joining the two centres of curvature in the ratio of the radii of curvature.

single ray of light,—the effect of a weak prism near the optical centre, and of a stronger and yet stronger one with every further removal from it, and this equally with concave and with convex lenses. It is evident, therefore, that by shifting a lens so as to bring a more peripheral part into use, a prismatic effect can be obtained just as much as if a prism had been combined with the lens in its former unshifted position. We can alter the position of the optical centre in one of two ways, the prismatic

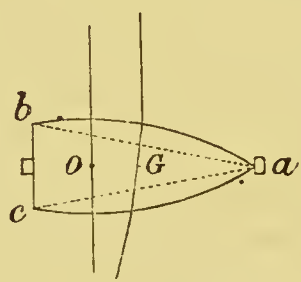

Fig. 38. A decentred lens.

equivalent of which is identical, but each of which possesses an advantage of its own. First, the lens may simply be *displaced* as it is, rim and all, by lengthening or shortening the spectacle-frame; or, secondly, the frame remaining unchanged, the lens can be *decentred* in its rim, as shewn in *Fig.* 38, The focal length of this lens is exactly the same as of that shewn in *Fig.* 39, inasmuch as its surfaces have the same radius of curvature, but it is as though it were cut peripherally, as shewn in *Fig.*

40, *a b* from a larger lens *c d*, so that its optical (*O*)

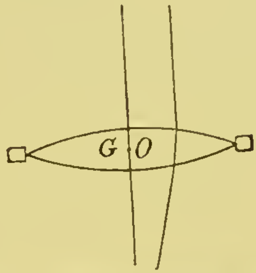

Fig. 39. A lens normally centred.

and geometrical (*G*) centres do not coincide as in an ordinary lens. A brief study of *Fig.* 38, which

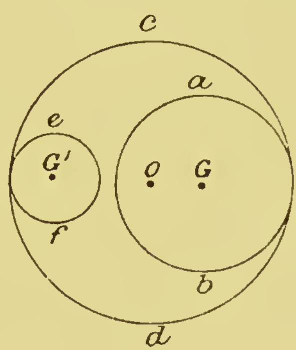

Fig. 40. Peripheric lenses cut out of a larger one.

gives the lens *a b*, of *Fig.* 40 in section, will show that the effect of cutting it eccentrically out of a larger lens is exactly the same as if a smaller lens of the same strength had been split to admit a prism between its two halves. Without altering its focal length then, a prism has been virtually introduced. The *geometrical* centre of a lens is the point midway between all edges, as *G* in *Figs.* 38, 39, and 40. The *optical* centre, on the other

hand, lies in the thickest part of a convex lens and the thinnest part of a concave one. For determining its position in any particular lens, methods will be described further on. A lens is said to be truly centred when the optical and geometrical centres coincide as in *Fig.* 39; but to be decentred when, as in *Fig.* 38, the optical centre (*O*) and the geometrical centre (*G*) are apart, the amount of decentration depending on their distance apart. The two points may be so far removed from each other that the optical centre may come to lie outside the lens altogether. Such would evidently be the case with the small lens *e f* in *Fig.* 40, since the optical centre is at *O*. *Fig.* 41 shews this small

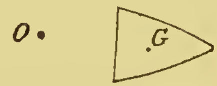

Fig. 41. A lens with its optical centre *O*, far outside itself.

lens in section, and it is evident at a glance that opposite surfaces have nowhere parallel tangents, and that one or both of the surfaces would need to be prolonged considerably before any point could be found in the one having a tangent parallel to that of a point in the other; were this done a line uniting the points would pass through the optical centre *O*.

We have seen that the effect of decentering a biconvex lens is equivalent to splitting it into two halves, and inserting a prism between them

(*b a c* in *Fig.* 38). The strength of this (virtually interpolated) prism depends on, first, the amount of decentration, and, second, the focal length of the lens. A strong lens needs decentering to a less extent than a weak one to produce the same effect. Table V. (p. 76) not only illustrates this but enables us to find the exact prismatic equivalent of decentering any lens. All that is required is to fix on the number of millimetres (in the highest horizontal row of figures) by which a lens is known to be decentred, and the prismatic equivalent is found in the column beneath, opposite the dioptric strength of the lens. Conversely, should it be required to know how much to decentre a given lens in order to combine with it a certain prism, the eye runs along the horizontal line of angles opposite the known strength of lens till that angle is found nearest to the strength of prism required, when the number of mm. at the head of the column will indicate the decentering necessary.

The word "decentering" is generally applied to making the optical centre no longer coincide with the geometrical centre of the lens, but we have already said that the prismatic effect is the same if the whole lens, rim and all, be simply *displaced*, as by lengthening or shortening the spectacle frame; in other words, it makes no prismatic difference, so long as the optical centre is moved, whether the geometrical one remains in *statu quo*, or whether

TABLE V.

	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	9 mm.	10 mm.	11 mm.	12 mm.	13 mm.	14 mm.	15 mm.	16 mm.	18 mm.	20 mm.	22 mm.	24 mm.	26 mm.	28 mm	30 mm.	32 mm.
·5D	1′ 43″	3′ 25″	5′	7′	8′ 25″	10′	12′	14′	15′28″	17′	19′	10′37″	22′27″	24′	26′	27′30″	31′	35′	38′	41′	45′	49′	52′	55′
·75D	2′ 30″	5′	8′	10′	13′	15′23″	13′	20′37″	23′	26′	28′	31′	33′	35′	38′	41′	45′	52′	57′	1° 2′	1° 6′	1° 12′	1° 17′	1° 22′
1D	3′ 26″	7′	10′	14′	17′	20′37″	24′	27′30″	31′	35′	38′	41′	45′	48′	52′	55′	1° 2′	1° 9′	1° 16′	1° 23′	1° 29′	1° 36′	1° 43′	1° 50′
1·5D	5′	10′	15′28″	20′37″	26′	31′	36′	41′	45′	52′	57′	1° 2′	1° 6′	1° 12′	1° 17′	1° 22′	1° 32′	1° 43′	1° 53′	2° 3′	2° 13′	2° 21′	2° 34′	2° 44′
2D	7′	14′	20′37″	27′30″	35′	41′	48′	55′	1° 2′	1° 9′	1° 16′	1° 23′	1° 29′	1° 36′	1° 43′	1° 50′	2° 4′	2° 18′	2° 32′	2° 45′	2° 58′	3° 12′	3° 26′	3° 40′
3D	10′	20′34″	31′	41′	52′	1° 2′	1° 12′	1° 22′	1° 32′	1° 43′	1° 53′	2° 3′	2° 13′	2° 24′	2° 34′	2° 44′	3° 6′	3° 26′	3° 46′	4° 7′	4° 27′	4° 48′	5° 9′	5° 29′
4D	14′	27′30″	41′	55′	1° 10′	1° 22′	1° 36′	1° 50′	2° 4′	2° 18′	2° 32′	2° 46′	2° 58′	3° 12′	3° 26′	3° 40′	4° 8′	4° 35′	5° 2′	5° 29′	5° 5[illegible]′	6° 24′	6° 50′	7° 18′
5D	17′	35′	52′	1° 9′	1° 26′	1° 43′	2°	2° 18′	2° 35′	2° 52′	3° 9′	3° 25′	3° 43′	4° 1′	4° 17′	4° 35′	5° 9′	5° 44′	5° 17′	6° 51′	7° 24′	7° 58′	8° 32′	9° 6′
6D	20′37″	41′	1° 2′	1° 23′	1° 43′	2° 4′	2° 21′	2° 45′	3° 5′	3° 26′	3° 46′	4° 7′	4° 27′	4° 48′	5° 9′	5° 29′	6° 10′	6° 51′	7° 31′	8° 12′	8° 52′	9° 32′	10° 12′	10° 52′
7D	24′	48′	1° 12′	1° 35′	2°	2° 24′	2° 48′	3° 12′	3° 37′	4° 1′	4° 24′	4° 48′	5° 12′	5° 36′	6°	6° 24′	7° 11′	7° 58′	8° 45′	9° 32′	10° 19′	11° 5′	11° 52′	12° 38′
8D	28′	56′	1° 22′	1° 50′	2° 18′	2° 46′	3° 12′	3° 40′	4° 8′	4° 35′	5° 2′	5° 29′	5° 55′	6° 24′	6° 5[illegible]′	7° 13′	8° 12′	9° 6′	9° 59′	10° 52′	11° 45′	12° 38′	13° 30′	14° 22′
9D	31′	1° 2′	1° 32′	2° 4′	2° 35′	3° 6′	3° 37′	4° 8′	4° 38′	5° 9′	5° 39′	6° 10′	6° 40′	7° 11′	7° 41′	8° 12′	9° 12′	10° 12′	11° 12′	12° 11′	13° 10′	14° 9′	15° 7′	16° 4′
10D	35′	1° 9′	1° 43′	2° 18′	2° 52′	3° 25′	4° 1′	4° 35′	5° 9′	5° 44′	6° 17′	6° 51′	7° 24′	7° 58′	8° 32′	9° 6′	10° 12′	11° 19′	12° 25′	13° 30′	14° 34′	15° 39′	16° [illegible]2′	17° 45′
12D	41′	1° 23′	2° 4′	2° 45′	3° 26′	4° 7′	4° 48′	5° 29′	6° 10′	6° 51′	7° 31′	8° 12′	8° 52′	9° 32′	10° 12′	10° 52′	12° 11′	13° 30′	14° 47′	16° 4′	17° 20′	18° 34′	19° 48′	21°
14D	48′	1° 35′	2° 24′	3° 12′	4° 1′	4° 48′	5° 36′	6° 24′	7° 11′	7° 58′	8° 45′	9° 32′	10° 19′	11° 5′	11° 52′	12° 38′	14° 9′	15° 39′	17° 7′	18° 34′	20°	21° 24′	22° 47′	24° 8′
16D	56′	1° 50′	2° 45′	3° 40′	4° 35′	5° 29′	6° 23′	7° 19′	8° 12′	9° 6′	9° 59′	10° 52′	11° 45′	12° 38′	13° 30′	14° 22′	16° 4′	17° 45′	19° 24′	21°	22° 7′	24° 8′	25° 38′	27° 7′
18D	1° 2′	2° 4′	3° 6′	4° 8′	5° 9′	6° 10′	7° 11′	8° 12′	9° 12′	10° 12′	11° 12′	12° 11′	13° 10′	14° 9′	15° 7′	16° 4′	17° 57′	19° 48′	21° 35′	23° 22′	25° 5′	26° 45′	28° 22′	29° 57′
20D	1° 9′	2° 18′	3° 26′	4° 35′	5° 44′	6° 51′	7° 58′	9° 6′	10° 12′	11° 19′	12° 25′	13° 30′	14° 34′	15° 39′	16° 42′	17° 45′	19° 48′	21° 48′	23° 45′	25° 38′	27° 29′	29° 15′	3[illegible]° 58′	32° 37′

TABLE V.* Shewing the prismatic effect of decentering any lens.

Procedure.—Find the dioptric strength of the lens in the left-hand column, the deviating angle of the virtual prism is then found opposite, under the number of mm. of decentering.

* This table is adapted from a thesis in 1884, where it appeared under another title ("Journal of Anat. and Phys.," Vol. XXI., p 32). It was made from the simple formula: Tan. $x = \frac{d}{f}$

x, being the deviating angle of the virtual prism; d, the decentering in millimetres; f, the focal length of the lens in millimetres.

it is shifted equally itself. How are we then to decide between these rival modes, in any given case? Decentering of any kind has the disadvantage of being attended with proportionate increase of weight, as much as if a prism were inserted: the lens of *Fig.* 38 is very evidently heavier than that of *Fig.* 39. This fault is entirely avoided by the alternative of displacing the entire lens (instead of only decentering the optical centre *in* the lens), but, on the other hand, such displacing entails, beyond certain limits, an unpleasant appearance and, worse still, a limitation of the field of binocular fixation. It may therefore be laid down as a practical rule, that the dislocation of the optical centre should be effected only slightly by displacing the whole lens, rim and all, leaving the remainder (should any more be required) to be obtained by decentration proper, that is, by decentering the lens *in* its rim.

To take an example, let us suppose it is desired to prescribe spectacles such that each optical centre is 10 mm. nearer to the median plane than the centre of the eye, and we have found that the centre of each eye is 32 mm. from the median (sagittal) plane. The effect of limitation of the field of binocular fixation must be considered for each case, for its inconvenience or otherwise depends on the distance of the patient's occupation; but, say that on consideration, 5 mm. of

displacing for each lens is all that is advisable; that will make each geometrical centre 27 mm. from the median plane, and leave 5 mm. for *decentering*.

In prescribing the spectacles, therefore, some such form as the following might be used:

	R.	L.
Spherical.	+ 4 D	+ 4 D
Geometrical centre.	27 mm.	27 mm.
Decentration.	5 mm. inwards.	5 mm. inwards.

The optician would have the choice of making these lenses by *decentration*, or by imparting sphericity to the surfaces of a *prism*. If by decentration he would grind a large lens, of the focal length ordered, find its optical centre (p. 147) and mark it with a dot of ink, then make another dot 5 mm. distant and cut out the lens with the last dot for its geometrical centre, placing it in a frame made with the centre of the rim 27 mm. from the middle of the bridge, so that the first dot (indicating the optical centre) shall lie to the inner side of the second dot. This plan is more accurate than that of making a prism, and is one to be encouraged. Whenever great precision is required, decentering should be chosen, unless indeed it be

combined with the manufacture of a prism, according to the second plan as follows. By Table V. the strength of prism required in the above prescription is easily found, for opposite 4D, and under the column 5 mm. we find 1° 10′. This is the deviating angle of the prism, and needs nearly doubling for the physical angle (p. 18).

After grinding, therefore, such a prism, and giving the prescribed spherical element to each of its surfaces, he would proceed to find the optical centre of the prismatic lens, by one of the methods described on p. 147, marking it with a dot of ink, and measuring the geometrical centre 5 mm. from it in the direction of the apex of the prism, to proceed thereafter just as before. In this way, any inaccuracy in the first manufacture of the prism is completely corrected. With weak prisms, indeed, combined with lenses, the whole problem of the difficulty of accurate manufacture would be solved in this way.

With lenses too weak, or prisms too strong, for the optical centre to lie in the combination at all, the prism-measure of the Geneva Optical Company comes to our aid. Placing a prismosphere* under the teeth connected with the index, and with its base-apex line parallel to them, it is easy to find the position which makes

* This is a term used by Dr. Percival in series with sphero-cylinder, to signify a "*prismatic lens*."

the index point to the required strength of prism. The central tooth then points to the future centre, which can be dotted with ink. The prismosphere should now be rotated 90° to make sure that when the tooth points to the dot, the index is at zero. This is to guard against the introduction of any *vertical* prismatic effect. If it does not point to zero, move the prismosphere laterally, and with care, till it does, when the new point marked out by the central tooth gives the true required centre.

When plain prisms are ordered, uncombined with lenses, we lose the power of adjusting their strength in this way.

In the case of cylindrical lenses, the effect of decentering depends on the direction in which it is made. Thus horizontal decentering of a lens, with a horizontal axis, has no effect, and the effect is greatest when the axis is vertical. With an oblique axis we need to find the focal length of the lens in the horizontal meridian. Here, though calculation would be easy,* the lens-measure of the Geneva Optical Company comes in most usefully to save us from it. By pressing it against the horizontal meridian of the lens, it records at once what we need to know. Adding this to the strength of the

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* To find the strength in dioptres, of the horizontal meridian of an oblique cylindrical lens, we need only, I believe, multiply the strength of the lens in dioptres by the sine of the angle of rotation of its axis from the horizontal.

spherical surface, if the latter be of the same sign, or subtracting one from the other if the signs be different, we then proceed as before.

As regards the distance of the optical centre from the median plane, we must remember that for a lens to have no effect on convergence, it must, at reading distance, be displaced 2 or 3 mm. more inwards than for distant vision. This position therefore forms the zero, by departing from which in either direction the visual axis is deflected.

As a rule, it is best for the geometrical centre to be in front of the centre of the eye in spectacles for distance, and 2½ mm. within this in spectacles for reading purposes.

To relieve deficient convergence, prisms should be set "edge out;" convex lenses should be displaced inwards, and concave lenses outwards.

To relieve excessive convergence, prisms should be "edge in," convex lenses displaced outwards, and concave lenses inwards.

"Orthoscopic Spectacles" as described by Dr. Scheffler, have been so long known that they must not altogether escape mention. In these, accommodation and convergence are affected in equal amount. The two lenses have but one optical centre common to both, just as though they were cut from opposite edges of one large lens. The proof of their correct manufacture is that they throw a *single* image of a flame upon a wall at their

focal distance. They have generally been prescribed in a few standard strengths, but they can be made for any patient by combining with each of his lenses a prism, whose deviating angle is found in Table V., opposite the description of lens, and under the number of millimetres by which the optical centre of each eye is distant from the median plane. These spectacles undoubtedly may have their place, but it is better to ascertain the conditions of convergence in each case and prescribe accordingly, rather than to adhere to any fixed combinations.

In the absence of a table, we may, as Ward Holden* suggests, remember that a lens of 1D, decentred 8·7 mm., produces the effect of an added prism of 1°. So that we may, if not wishing great accuracy, multiply 8·7 mm. by the number of any prism we wish to obtain the effect of, and divide by the number of dioptres of the lens. Thus :—Required the decentering necessary in a lens of +3D to produce the prismatic effect of 4°?

Ans. $8{\cdot}7 \text{ mm.} \times 4^\circ \div 3 = 11{\cdot}6 \text{ mm.}$

In working with this rule, it is the refracting angles of prisms that are dealt with. Table V. gives the deviating angles. To produce the effect of a prism of 1°*d*, a lens of 1D must be decentred 17·45 mm.

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* Archives of Ophthalmology, Vol. XX., p. 21.

Chapter XI.

THE STUDY OF CONVERGENCE.

To prescribe prisms satisfactorily it is necessary to know something of the physiology of convergence, our knowledge of which, however, is still very incomplete. According to Hering's well-proved theory, both eyes move together as though they were a single organ. No ocular muscle can in health receive a nervous impulse without the transmission of an equal impulse to some associated muscle of the other eye. Thus there is one conjugate innervation which turns both eyes to the right, and another which turns both eyes to the left. Besides these lateral movements, there are, in all probability, two other innervations, which turn both eyes respectively upwards and downwards. By these innervations, which need not detain us, the visual axes can be simultaneously moved in any direction, but were there no other innervation, we would see all near objects double, from inability to converge the visual axes upon them. This want is supplied by the innervation of convergence, which controls the internal recti, one as much as another, and independently of their control by the conjugate lateral innervations. The efforts of convergence and

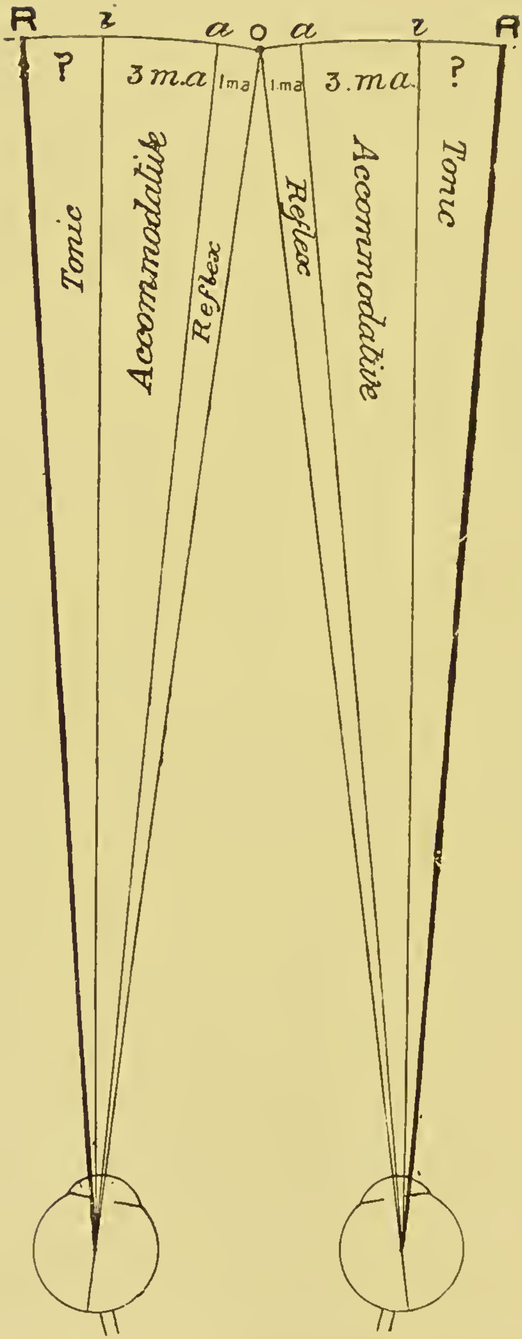

Fig. 42.—The three grades of convergence in vision for ·25m.
To scale: half life size.

accommodation are intimately intersusceptible, so that the slightest impulse to either makes a difference in the other, albeit the difference is not necessarily equal in amount. If one eye be covered while the other is fixing a near object, the occluded eye will, in the majority of persons, deviate outwards 3° or 4°. This behaviour of a normal eye, placed in the dark, was some years ago noted by the use of the blind spot.* V. Graefe had long before shewn how by placing a prism, edge up, before one eye, and looking at a dot on paper with a vertical line through it, he could detect what he called "muscular insufficiency" in many cases of myopia. But his test did not appear to reveal that a lesser degree of the same thing was a physiological condition. *Fig.* 43 shews a slight modification of his

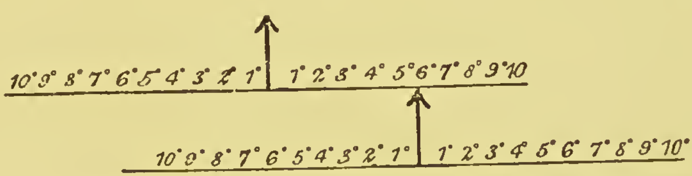

Fig. 43.—A tangent scale reduplicated by a vertical prism.

test, the results of which agree with the blind spot method. It consists of a lithographed scale, graduated in tangents of degrees to right and left of a central zero, from which rises a vertical arrow. The

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* "On a new method of determining the relation between Convergence and Accommodation of the Eyes." Trans. Oph. Soc., Vol. III., p. 290.

scale is held ·25m. before the eyes, and is reduplicated by a square prism of 12°, held vertically before one eye. The lower arrow then points upwards to that number which measures the deviation of the eye. We call this a "*latent* deviation," because it does not exist in ordinary vision, being overcome by a subtle visual-reflex action which protects us from seeing double. This action is discontinued when the false image produced by a prism is so far removed from the true one as to make it hopeless to attempt their union by overcoming the prism (*cf.* p. 63). The latent deviation revealed by this test shews that the association of convergence with accommodation is not complete centrally, so that a supplementary action is needed to supply the deficiency, and since the involuntary motive for this action is afforded by the desire to fuse two images into one, the proportion of convergence thus kept up may be called the "fusion supplement," or, better still, the "reflex convergence."

In ordinary binocular vision there is perfect concert between accommodation and convergence, so that the two visual axes meet exactly at whatever point is accommodated for, and the visual-reflex is maintained in activity by the fact, of which some cerebral centre is every moment kept sensible, that were the effort abated, the united mental images would immediately resolve them-

selves into two. To keep them from doing so, the joint sensations in the brain must all the while be bearing between them the message of continually impending (yet as quickly averted) double vision, by threats of double images, so slight and frequent, that they produce the required effect without our being conscious of their existence. It is difficult to conceive the exquisite mechanism at work when we remember that, if double images are produced artificially, or by disease, it is usually impossible for the mind to tell to which eye each image belongs —whether, therefore, the visual axes are crossed or not, and whether convergence needs to be increased or relaxed to bring the images together.

I have purposely dwelt at length upon this reflex, because an action so complex must necessarily be more tiring than the mere overflow of one impulse into another. If, therefore, there be an undue proportion of reflex convergence, there is a waste of co-ordinating nervous energy. In many cases this waste is of no consequence, but in others it may give rise to the so-called "muscular asthenopia" of V. Graefe, which, however, is really in many instances a *central asthenopia*, though there may sometimes be a muscular element as well. It is just, I believe, as as in "writer's cramp," and the various "trade pains," where it is not so much the muscles that get tired, as the co-ordinating apparatus. Asthenopia from this cause can be relieved by prisms with their

edges out, which lessen the convergence necessary for single vision. It is of course desirable to only relieve a sufficient portion of the reflex convergence, and not the whole of it. Take the example of an asthenopic myope, with a latent deviation of 10° at reading distance; this shews that the fusion supplement for *each* eye is 5°, for since *either* eye deviates outwards on exclusion, the deviation must be equally divided between them. The normal fusion supplement is nearly 2° for each eye, so that we have to deal with 3° of excess. Should we correct the whole of the excess with prisms? I think not, for we may assume that a myope has, by long habitude, acquired a toleration for a larger amount of reflex convergence. Prisms to relieve half the excess would probably meet the case. It is taken for granted that the test for deviation is made with the reading lenses on, otherwise the deviation would be greater still. We may of course lessen the deviation by increasing the strength of the reading lenses if these are not full strength, but this throws the strain on accommodation instead, and, in people who are not young, is not always well borne. It is, in any case, too, only practicable within the limits of full correction.

Endless variations are met with, so that it is better to study the principles of convergence than to make rules of too arbitrary a nature. High latent deviations may occur in other conditions of refrac-

tion than myopia, but in every case in which prisms are ordered to correct divergence in near vision, care should be taken that they are not also used for distant vision, unless indeed the distant divergence be great enough to indicate the same prisms for itself. Sometimes, though much more rarely, there is, even in near vision, a tendency to excess of convergence, instead of deficiency. In this case convergence has to be lessened (inhibited?) in order to see single. Now, therefore, the reflex convergence is negative instead of positive. In some of these cases it is convex lenses that are indicated more than prisms. When the latter are ordered, their edges should be inwards. Latent convergence at reading distance is naturally much rarer than latent convergence in distant vision. Spasm of convergence occurs independently of hypermetropia, but its causes, supposed sometimes to be hysterical or reflex, are little understood. The influence of helminthiasis in this direction, however, is undoubted.

Initial Convergence.—Of the three elements of convergence this is the first, and is shewn under the name of "tonic convergence" in *Fig.* 42. We must first ask "What is the *starting point* of convergence?" We do not exactly know what position the eyes would take in the absence of any converging innervation, though the divergence which follows monocular amblyopia seems to shew it

would be one of considerable divergence. Were all the innervations to cease, the anatomical position of rest of the eyes would undoubtedly be one of considerable divergence, as Hansen Grut ably maintained in his Bowman* lecture. Le Conte† shewed that during sleep, and even, in his case, during drowsiness, the eyes diverge, as they also do in drunkenness, under chloroform, and at death. In ophthalmoplegia externa, divergence also occurs. The ocular muscles no doubt possess a physiological tone, similar to that of the other skeletal muscles, but what influence it has on the position of the eyes it is difficult to decide. In addition to this common muscular tone there is a persistent activity of the converging innervation, which disappears when deeply under chloroform, but which in ordinary conditions prevents the eyes from assuming their position of anatomical divergence. Mr. Berry views it as "the tendency to persistence of a constantly called for state of innervation," and Hansen Grut aptly compares it to the tonic element in the accommodation of a hypermetrope. By this tonic convergence, the visual axes are brought to practical parallelism, so that on viewing a distant object, and occluding one eye, it either remains undeviated, or only aberrates slightly, more

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* Trans. Oph. Soc., Vol. X., p. 1.

† Le Conte on "Sight," p. 254.

often in the direction of divergence than convergence in emmetropia, and more often in the direction of convergence than divergence in ametropia. In the first edition of this book, slight convergence of the visual axes was mentioned as the prevalent latent condition in distant vision, that having been found in the comparatively few cases examined by a difficult and slow method. The conclusions in the text are from an investigation among the students attending Mr. Berry's clinic, by the glass rod test, which enables a large number to be tested with rapidity and accuracy, and they are recorded by Mr. Berry in the "Trans. Oph. Soc.," vol. xi., p. 160. If we accept, as I now believe we must, divergence instead of parallelism, to be the anatomical position of the eyes, the diagram in which I once represented three grades of convergence,* must be modified accordingly into the form shewn in *Fig.* 44. The strong lines (*R*, *R*) indicate the supposed position of the visual axes were all nervous impulse abolished. The lines *i*, *i* indicate the initial convergence during waking hours, due partly to muscular tone, and partly to involuntary tonic action of the converging innervation. It is this position of the visual axes (*i i*), to which they are brought by tonic convergence, that is noticed when we use V. Graefe's "distant equilibrium test," or

* Oph. Rev., Vol. V., p. 345.

any of its successors, such as the double prism or glass rod. Latent divergence in distant vision indicates a deficiency, and latent convergence

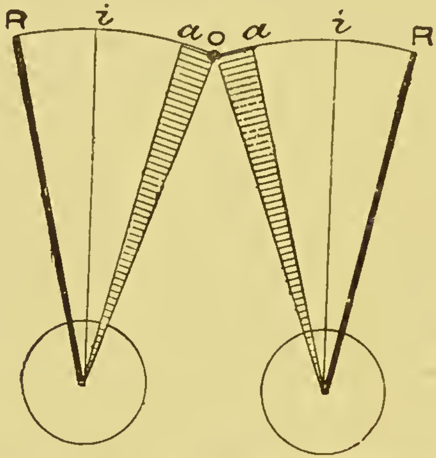

Fig. 44.—The three grades of convergence.

an excess, of *tonic convergence.* In either case the aberration is corrected, when both eyes are in use by reflex convergence, positive or negative, as the case may be.

Excess of tonic convergence is sometimes apt to occur temporarily after prolonged use of the eyes in near work, also more permanently in a certain proportion of those myopes, who hold their work near their eyes, and yet retain binocular fixation. It occurs, too, in most hypermetropes if uncorrected by lenses, and in a few even when corrected. The so-called spasm of convergence, from reflex and hysterical causes, already referred to, is probably a simple excess of tonic convergence.

If found to be persistent, and suspected to give rise to asthenopia, excess of tonic convergence may be relieved by prisms with their edges in, of a strength that will only partially correct the anomaly.

Deficiency of Tonic Convergence, evidenced by latent divergence in distant vision, is rather common, and appears to be sometimes due to reflexes through the fifth nerve, especially from carious teeth; also to want of nervous tone in the system generally. It is, however, found besides in many who are free from either of these conditions, and in a class of myopes, especially those who read with one eye. It may, if persistent, and if there be any indication for treatment, be relieved by prisms with their edges out, to correct a portion—say a half—of the defect. I think it would be a useful nomenclature for prisms to designate those with edge in as *plus*, and those with edge out as *minus*, just as we speak of plus and minus lenses. Thus -3°d. would mean a prism, deviating 3°, edge out, and $+5^\circ$P. would mean a prism with physical angle of 5°, edge in. It is so easy to remember that plus conditions of convergence are relieved by plus prisms, and minus conditions of convergence by minus prisms. In normal eyes, plus prisms increase the convergence, and minus prisms lessen it; this, too, is easy to remember.

ACCOMMODATIVE CONVERGENCE.—In distant

vision, as we have just seen, there are only two grades of convergence, the tonic and the reflex, shewn in *Fig.* 45. In near vision there is an intermediate

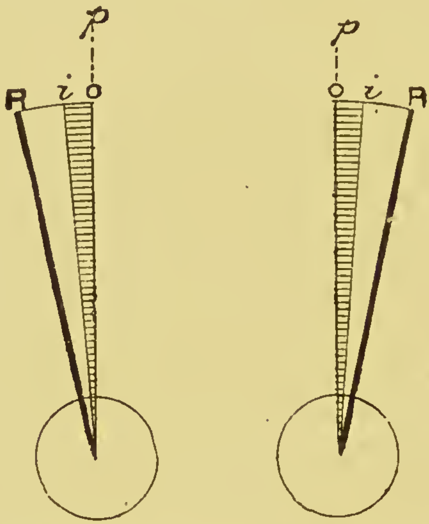

Fig. 45.—The two grades of convergence in distant vision. Tonic *C* is *Ri*, and Reflex *C* is *io*.

grade—the accommodative. If one eye be occluded by the hand while vision is directed, first to a distant object, and then to a near one, the occluded eye deviates inwards under the hand from an impulse to convergence, which is due chiefly to sympathy with accommodation, but also to the habit of converging when attention is directed to a near object (Grut's "Nahebewusstein"). The second grade of convergence is therefore added on to the first or tonic grade, and its amount depends, of course, on the amount of accommodation in exercise. As a rule,

each dioptre of accommodation is accompanied by about three-quarters of a metre-angle of associated convergence, so that in a typical emmetrope, the 4 D of accommodation in exercise for vision at a quarter of a metre, are accompanied by 3 m.a. of convergence, leaving a deficit of 1 m.a. to be made up reflexly, as shewn in *Fig.* 42. To investigate the accommodative convergence at different distances, I have made scales, similar to that of *Fig.* 43, for use at 1m., ·5m., ·33m., and ·25m., but for practical purposes we need only use the last.

Now we may notice some of the conditions which affect the amount of accommodative convergence. Whatever paralyses the ciliary muscle locally, such as, *e.g.*, weak atropine drops, renders the ciliary muscle less responsive to its motor impulses, thereby increasing the *effort* of accommodation, and with it the associated convergence. If, even without such paralysis, the object fixed be approached to the "punctum proximum" of accommodation, the accommodative effort becomes so much greater than the effect produced in the lens, that the associated convergence exceeds all proportion, and produces a latent convergence. In hypermetropia, the accommodative convergence, without correction of refraction, is, of course, greater as a rule than in emmetropia.

What conditions *lessen* the accommodative convergence? Any which make accommodation easier.

so that work is done with less effort. Myopia, for instance, renders accommodation unnecessary in vision beyond the far point. *Figs.* 46 and 47

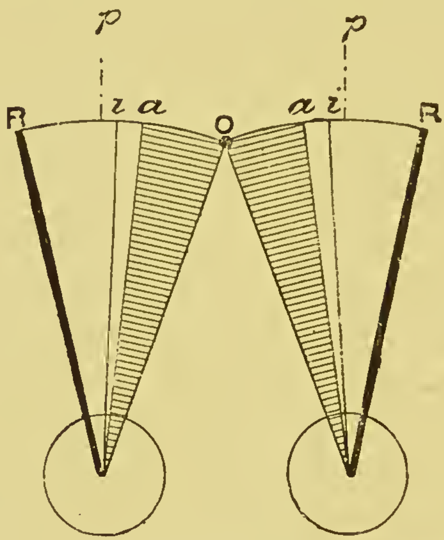

Fig. 46.—The conditions in a myope who has latent convergence in distant vision.

shew how the diminution of accommodative convergence (*i a*) in myopia is made up by greater reflex (and perhaps also voluntary) convergence (*a o*), the amount of which is influenced by the condition of the tonic convergence (*R i*) which is excessive in *Fig.* 46, and deficient in *Fig.* 47. To observe the effect of *eserine*, which, in weak solution, renders the ciliary muscle more excitable to stimulation without producing spasm or myopia, I placed gr. $\frac{1}{1000}$ within each lower lid. This allowed me to retain full distant vision, but increased my fusion supplement at 10 inches, from one to two metre angles.

The effect on tonic convergence, as shewn by the distant equilibrium, was practically *nil.* This experiment shews how truly convergence is affected, not by accommodation, but by accommodative

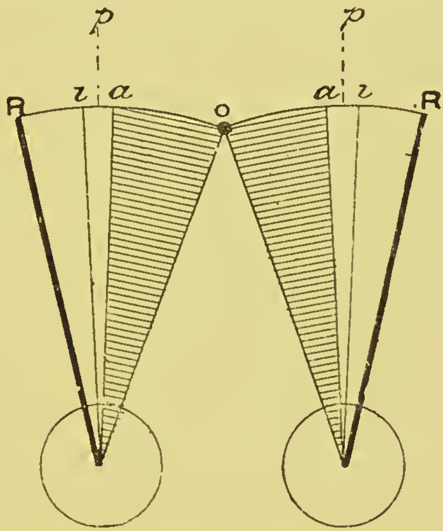

Fig. 47.—The conditions of a myope who has latent divergence in distant vision.

effort, for accommodation was the same both with and without the eserine, but with it, the effort was less, and the impression on convergence reduced in consequence.

REFLEX CONVERGENCE.—This, the third grade of convergence, has already been noticed in part. It is the element most affected by ocular fatigue, as is well seen in cases of "periodic squint," as they are called. On rising in the morning, there is perhaps no squint, but as the day wears on, and the eyes get tired, the squint appears, its mani-

festation being partly* due to the diminishing vigour of the visual-reflex, the amplitude of which varies with the nervous vigour of the moment. As soon as the amplitude of the visual reflex becomes less than the squint, the latter is no longer overcome.†

Fig. 48 represents a case of true periodic squint

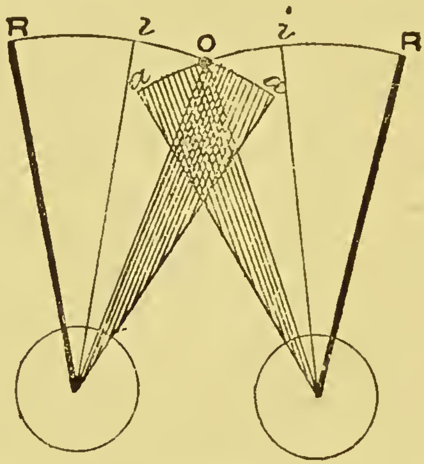

Fig. 48.—A periodic squint in its latent phase.

in the early morning. The first two grades of convergence, each excessive if the patient be a

* I say "partly," because I believe that the converging centre may perhaps become more excitable by the constant stimulus of accommodation in these cases.

† These cases must not be confused with other groups—such as "accommodative squints," in which the hypermetropia may be absolute, or accommodation is partially surrendered, except on paying close attention to an object, or cases in which the squint appears only in near vision, because accommodation there reaches the limits in which effort becomes disproportionate to work.

hypermetrope, would bring the visual axes to *a a*, but they are brought back to *o* by negative reflex convergence (shaded area). As the day wears on the effort is abandoned, though in distant vision the squint may perhaps still be overcome. By correcting the hypermetropia we lessen the accommodative convergence, and the tonic convergence will thus gradually tend to get less, the excitability of the converging centre being no longer kept up by stimulation from inordinate accommodation. It is thus that squints are sometimes cured by spectacles in the course of years. The effect of tenotomy on the same case is shewn in *Fig.* 49. By increasing

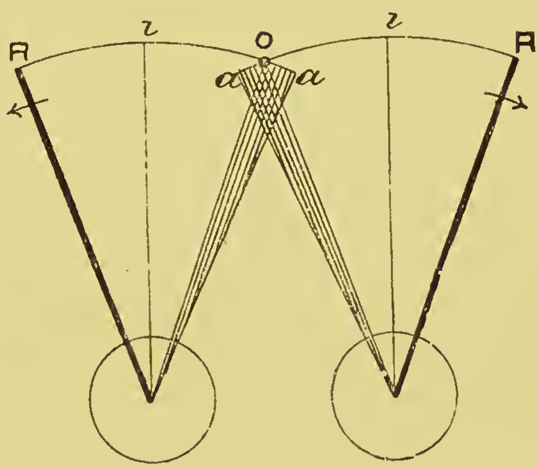

Fig. 49.—The same case as in *Fig.* 48, but relieved by tenotomy which displaces the anatomical position outwards.

the anatomical divergence it allows more room for the excessive tonic and accommodative convergences. The amplitude of the visual reflex can often be increased by nerve tonics, such as Fellows'

Syrup, rest, and change of air. To measure how great the amplitude of visual-reflex is, we may

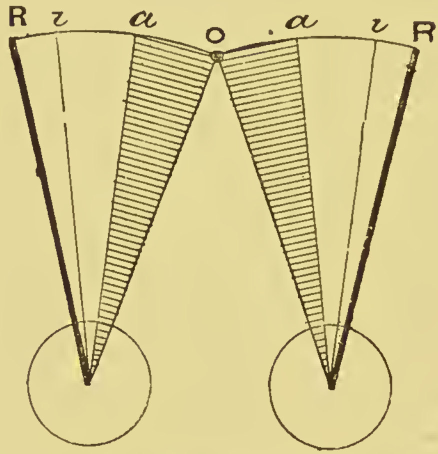

Fig. 50.—High fusion supplement (*ao*) from deficient tonic convergence (*Ri*).

use the tests described on p. 120. Since the third grade of convergence is complementary to the

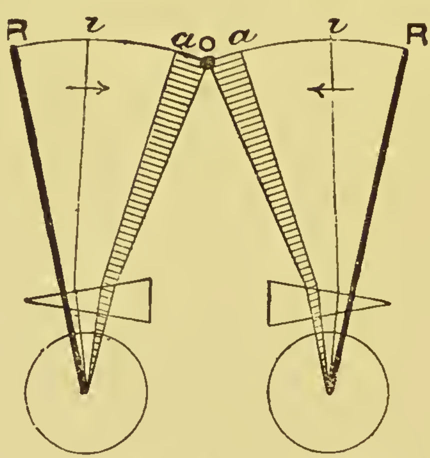

Fig. 51.—Same case as *Fig.* 50, relieved by prisms.

other two, its amount can be altered by altering them. In *Fig.* 50, for instance, the tonic conver-

gence is deficient, the accommodative convergence normal, and the reflex convergence, therefore, excessive. The relief of this by prisms is shewn in *Fig.* 51, and by tenotomy in *Fig.* 52. The prisms do not alter the anatomical position of the eyes

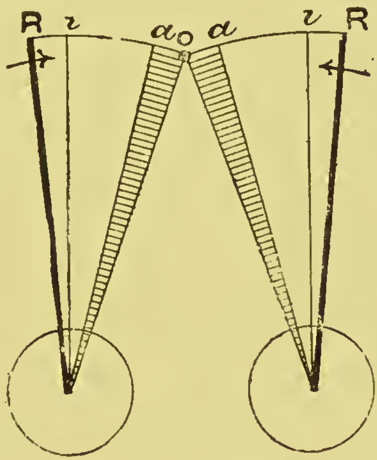

Fig. 52.—Same case as *Fig.* 51, relieved by tenotomy.

(*R*, *R*), or alter the accommodative convergence, but they alter the "initial position" (*i*, *i*) which the eyes assume in consequence of tonic convergence. We can alter

The *anatomical* position (*R*, *R*) by *tenotomy*,
The *initial* position (*i*, *i*) by *prisms*,
The *accommodative* convergence by *lenses*,
The *reflex* convergence by any of the three.

EFFECT OF TRAINING.—This is a very important subject, and needs working out more thoroughly. By wearing prisms we can alter the position of equilibrium of the eyes for some little time after-

wards. Thus, if I wear adducting prisms, amounting together to 11°d, for ten minutes, the rod test reveals, on their removal, $5\frac{1}{2}$° of latent convergence as the position of equilibrium for distance instead of my usual convergence of only $\frac{1}{2}$°. In other words, the tonic convergence has been temporarily increased by 5°, and takes a good many minutes to recover its usual dimensions. In near vision, under the same conditions, the accommodative convergence is shewn to have been augmented by 7°. The experiment causes slight headache, not so much during the wearing of the prisms as afterwards, and especially when engaged in near vision, on account of the fact that the customary *reflex adduction* is replaced by the unwonted requirement of *reflex abduction.*

Again, if I wear convex lenses for near vision for a few hours, the latent divergence at reading distance is, after their removal, less than before.* This is because of the training which the converging innervation has undergone in the increased *relative* demand made upon its energies. It explains why spectacles, causing discomfort at first, become tolerated after a while. There are, however, limits to the effect of training, otherwise prisms would never be needed, except for pareses of ocular muscles. If there were no limits to training, there would never

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* Syme Essay, 1884.

be a concomitant squint, for the training of the whole life, before the squint appears, is in the direction of overcoming it. The complex process of co-ordination of course requires definite cells and groups of cells (centres) for its seat, but the mutual adaptation and correlation of these, quantitatively, seems largely left to be perfected by *education*. As I have said elsewhere, "Circumstances cannot create a faculty, however much they may develop or retard its exercise, but we can conceive that faculties were created with a view to circumstances, and even capable, within limits, of being modified by them."* The human body is thus made capable, within limits, of adapting itself to its circumstances, and in no part of it is this more beautifully seen than in the relation between convergence and accommodation of the eyes. In neurasthenics the limits of adaptability may be more quickly reached, or adaptation may be only attained at the cost of much discomfort, while even in healthy individuals unusual conditions may make it better to not trust to it altogether, but to meet the case with prisms. Cases which require prisms are generally those in which power of adaptation has already signally failed.

It is under the heading of "training" that I should mention Dr. Dyer's so-called "invigorant

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* Journ. Anat. and Phys., Vol. XXI., p. 21.

plan" of treating latent deviations, for it does not, if my belief be correct, invigorate the muscles, but simply trains the efforts of accommodation and convergence to assume broader relations to each other in their work. It consists in giving the patient four squared prisms, of about $2\frac{1}{2}°$, $5°$, $10°$ and $15°$ respectively. Twice a day for ten minutes he is to fix a flame, or door knob, at 20 feet, and exercise abduction and adduction, beginning with the weakest prism, and mounting the ladder to the strongest. If these exercises strengthen anything, it is the visual reflex, the amplitude of which they increase, and by so doing increase the relative range of convergence. Though I have not yet tried this, I can quite believe a case might now and then be found suitable for its trial. I would, however, believe Dr. Dyer's plan to promise greater success in a modified form, when instead of being used to enlarge the powers of the visual-reflex, it might tend to induce a new relationship between the efforts of convergence and accommodation. If, for instance, in a case of latent divergence, weak *plus* prisms (*i.e.* adducting) were worn for half an hour twice a day, say after breakfast and after dinner, or whenever the nervous conditions are strongest, would not the latent squint gradually disappear in some cases? I believe it might, though I cannot speak from experience. Since this plan would be better with vision for all dis-

tances, ordinary occupations could be pursued while the training prisms are worn.

It cannot be denied that prisms given to relieve a defect tend to stamp permanency upon that defect, therefore they should never be given without ample reason. *The great majority of latent deviations call for no treatment at all.* Some cause trouble, and should be treated. Constitutional treatment may suffice for some cases. If training is feasible it may be tried. If training cannot be borne without headaches or much discomfort, relieving prisms may be ordered, but such as to under-correct the anomaly. Tenotomy, or advancement, should only be resorted to in a small number of cases, where the deviation causes undoubted trouble, and is too great to be relieved by prisms. It is rarely justifiable, in my opinion, for a latent deviation of less than 6° in distant vision, and often unnecessary even then: all depends on the history and symptoms. It remains to be found out what proportion, and what class of cases can be cured by training. It may be that some cases of hyperphoria which we now relieve by correcting prisms could be "trained away," so to speak, by periodic use of adverse* prisms, but my impression is that many would not tolerate the training process, and it is

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* By adverse prisms, I mean prisms set with the apex in the opposite direction from relieving prisms.

certainly a great pleasure to afford relief sometimes to the symptoms of years by "relieving prisms," even though we know the defect is not really cured.

NOTE.—Strictly speaking, there are four elements of convergence, though the first and third are perhaps closely related. The four are: (1,) Tonic; (2,) Accommodative; (3,) Convergence due to "knowledge of nearness," or in other words, "Voluntary convergence," for we cannot, without special practice, converge the eyes voluntarily, under ordinary conditions, without doing so by thinking of a near object; (4,) Fusion convergence. Of these four elements I have included the second and third under the one name of "accommodative convergence," to simplify practical work. On looking at a near object, the voluntary and accommodative elements of convergence bear very different proportions to each other in different individuals. It is possible that when some persons direct their vision to a near object the voluntary impulse passes chiefly into the channel of convergence, accommodation being more secondary, while in others it passes chiefly into the channel of accommodation, convergence being more secondary. In the former class we should expect the latent position of the eyes to be less influenced by changes in accommodation than in the latter class.

Chapter XII.

THE EFFECT ON THE FIXATION LINES OF DISPLACED OR DECENTRED LENSES.

This study will cause embarrassment if allowed in any way to be confused with those of previous pages. It must be clearly borne in mind that the prismatic equivalent of decentering a lens, and the effect of that decentred lens on convergence, are two distinct things, though at first sight they appear the same. Suppose we decentre a lens, or, what is the same thing, suppose we place a normally centred lens precisely before one eye and associate a prism with it. It has hitherto been supposed that the angle of deviation of this prism would exactly express the effect on convergence. But it is not so. *Fig.* 53, *B*, represents a decentred lens, the action of which is precisely similar to a prism and lens, for, as said before, it is as if the lens were split in two, and the prism shewn in dotted outline were inserted between the two halves. Now it will be evident at a glance, that the angle *c d f* is the deviating angle of the virtual prism, while the greater angle, *d c e*, is that which

represents the effect on convergence. It makes no difference to the effect on convergence whether a

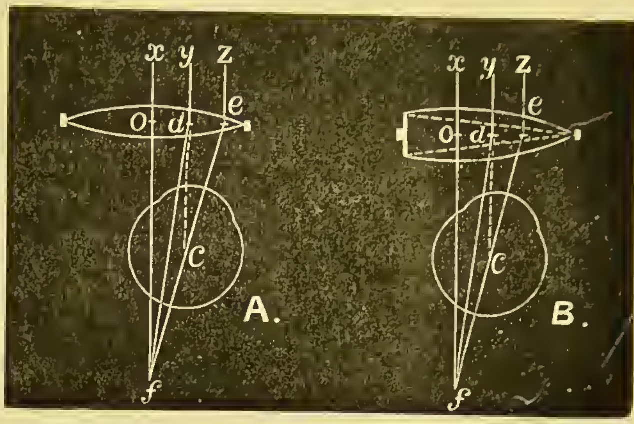

Fig. 53.—*A*. The right eye is looking at a distant object through a lens *displaced* inwards, *rim and all*, by a distance *o d*; *d c e* is the effect on convergence. *B* is the same eye looking through a lens of the same focal length, but *decentred in* its rim by the same distance *o d*. The dotted lines indicate the prism that would be equivalent to the decentering, and the effect on convergence is the same as with *A*.

lens is decentred in its frame, as in *Fig. B*, or displaced, frame and all, to the same amount, as in *Fig. A*.

There is one exception to the discrepancy between the prismatic equivalent and the effect on convergence, and that is when the lenses are *convex* and the object of fixation is at their focal length, and immediately in front of the eye. Within that distance the effect on convergence is less, and beyond it more, than the prismatic equivalent. But with *concave* lenses it is always *less*. In the first edition I went no further than to point out these

facts, though in a former essay* I had given formulæ for the effect of decentering on the visual axis in distant vision, and a series of simple rules for both near and distant vision, which may now be reproduced in a slightly altered form.

If a line (not represented in the figures) be dropped from c (the centre of rotation of the eye), perpendicularly upon the principal axis ($o\,f$) of the lens, the length of this perpendicular equals the decentration of the lens ($o\,d$). It also cuts off from the line $o\,f$, which measures the focal length of the lens, a portion equal to $d\,c$.

Therefore $$\text{Tan. } o\,f\,e = \frac{o\,d}{o\,f - d\,c}.$$

But $o\,f\,e$ is equal to the angle $d\,c\,e$ which expresses the effect of the decentred lens on the visual axis, and which we may represent by x. Let d stand for the decentering ($o\,d$), and f for the focal length of the lens in mm. ($f\,o$), and let the distance of the lens from the centre of rotation of the eye be 25 mm. Then, with a convex lens

$$\text{Tan. } x = \frac{d}{f - 25}$$

and with a concave lens,

$$\text{Tan. } x = \frac{d}{f + 25}.$$

In these formulæ,† the distance of 25 mm. for

* Journ. Anat. and Phys., Vol. XXI., p. 36.
† Journ. Anat. and Phys., Vol. XXI., p. 35.

the distance of the lens from the centre of the eye was chosen because it allowed some simple rules to be made for forming an approximate estimate of the effect of lenses on the visual axes without the use of tables, or tangents, as follows. They are for distant vision only as they stand, but when modified, as explained further on, they suffice also for near vision.

I.—*To find the effect on the visual axis of combining a lens and a prism, or of decentering a lens.*

(*a.*) *With a + lens. Rule: Divide the deviating angle of the prism* (in the case of decentering the *virtual* prism can be found from Table V.) *by the number of inches in the focal length of the lens* LESS ONE. This, *added* to the deviating angle of the prism, gives the effect on the axis of fixation. *Example:* Given a $3^\circ\,d$ prism and a + 10D lens, required the effect on the visual axis. A + 10D lens has a focal length of four inches. This, *less one*, is three inches.

$$\frac{3^\circ}{3} = 1^\circ. \quad \text{Add this to } 3^\circ = 4^\circ.$$

The effect on the visual axis, therefore, is 4°.

(*b.*) *With a – lens. Rule: Divide the deviating angle of the prism* (or of the virtual prism from Table V. in the case of decentering) *by the number of inches in the focal length of the lens* PLUS ONE. This, *deducted* from the deviating angle of the prism, leaves the effect on the axis of fixation.

Example: Given a $3° d$ prism, and a $-10D$ lens, required the effect on the fixation line. The focal length of a $-10D$ lens is four inches, which *plus one* is five inches. The deviating angle must therefore be divided by 5, thus:

$$\frac{3°}{5} = 36'; \quad 3° - 36' = 2° \ 24'.$$

The effect on the visual axis is, therefore, $2° \ 24'$.

II.—*To find what prism we must combine with a lens, in order to produce any required effect on the axis of fixation.*

(*a.*) *With a + lens. Rule: Divide the required effect on the fixation line by the number of inches in the focal length of the lens.* This, *deducted* from the effect required gives the prism that will produce it. *Example:* Required with a $+10D$ lens to deflect a fixation line by $4°$. What prism is needed? The focal length of the lens is four inches, $4° \div 4 = 1°$. Deduct this from $4° = 3°$. The prism required is $3° d$. Or the decentering required is (from Table V.) a trifle more than 5 mm., since a lens of $10D$ decentred to that amount has a prismatic equivalent of $3° d$.

(*b.*) *With a − lens. Rule: Divide, as before, the required deflection of the axis by the number of inches in the focal length of the lens.* This, *added* to the deflection required, gives the prism that will produce it. *Example:* Required with a $-10D$ lens to deflect a visual axis by $4°$. What prism is needed?

The focal length is four inches. $4^\circ \div 4 = 1^\circ$. Add this to $4^\circ = 5^\circ$. The prism required is $5^\circ\, d$.

From Table V. it is seen that a lens of 10D must be decentred nearly 9 mm. to have the effect of a prism of 5°.

These rules may prove of value in the correction of hyperphoria, where considerable exactness is required. The examples given illustrate well how much less deflection is imparted to the line of fixation by decentering a concave lens than a convex one. Thus, we have seen that to produce a deflection of 4°, a convex 10D lens needs a decentering of 5 mm., and a corresponding concave lens 9 mm.

For near vision the rules require a little modification. Instead of reading the "focal length" of the lens we should read the "conjugate focus." They then apply in near vision with quite sufficient accuracy for practical purposes, for though it is true that they suppose the object of fixation to be in front of each eye, when it is really in the median plane, this only introduces a trifling error, and it must be remembered that no fixed rules or tables are quite accurate clinically, since the interocular distance, and the distance of the lenses from the eyes, vary so greatly in different patients.

To find the conjugate focus of a lens we *subtract* from the reciprocal of its focal length, the reciprocal of the distance of the object from the lens. Thus,

with the object at 12 in., a +10D lens (whose focal length is therefore 4 inches) has a conjugate focus of ($\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$) 6 inches.

Since a concave lens has a virtual focus, it is prefixed by the negative sign, and therefore we *add* the reciprocals; thus, with a −10D lens under the same conditions as before, the conjugate focus is ($-\frac{1}{4} - \frac{1}{12} = -\frac{1}{3}$) 3 inches.

Some may perhaps find a difficulty in using these rules, and fortunately the subject has recently been followed up by Dr. Percival, who is an able mathematician, and has drawn up a series of tables from a formula which assumes reading distance to be $\frac{1}{3}$ metre, the distance between the ocular centres to be 64 mm., and the distance of the lenses from the centres of rotation of the eyes to be 27·4751 mm. This distance is chosen since it is that at which lenses should theoretically be placed in order that the size of images formed on the retina may be the same as in emmetropia: it is, in short, the distance of the anterior principal focal plane of the eye from the centre of rotation. They are made for distant vision from the formula, similar to my own,

$$\text{Tan. } \chi = \frac{d}{f-k}$$

Where χ = deflection of visual axis.
f = focal length of lens.
k = distance of lens from centre of rotation.

For near vision I had made no formula, and Dr. Percival has supplied the deficiency by the following:

$$\text{Tan. } \chi = \frac{dp - fm}{fp - k(p-f)}$$

This is for a convex lens; for a concave one the signs must be reversed, being plus instead of minus, since a negative value must be given to *f*.

It will be noticed that in the near vision table the prismatic equivalents differ from those of the distant vision table by a constant quantity, viz., 5′ for each degree, and 10′ for each metre angle. I give Dr. Percival's own examples to illustrate their use.

EXAMPLES ILLUSTRATING THE USE OF THE TABLES.

TABLE VI.—A hypermetrope of +8D and esophoria 1 ma. at 6 metres will have this convergence defect corrected by decentering the 8D lens 3·1 — mm. outwards, or, what comes to the same thing, by associating it with a prism of 1° 26′ *d* (not 1° 50′).

TABLE VII.—A patient requiring +12D glasses for reading, who can only maintain convergence for a distance of ½ metre (2 ma.), must have his glasses decentred 4·6 mm. inwards.

TABLE VIII.—A myope requiring −6D for reading, who can only maintain 2 ma. of convergence, must have his glasses decentred 4 mm. outwards, or

DR. PERCIVAL'S TABLE FOR DISTANT LENSES.

TABLE VI.

				1 D	2 D	3 D	4 D	5 D	6 D	7 D	8 D	9 D	10 D	12 D	14 D	16 D	18 D	20 D
CONVEX.	Divergence decentration inwards. Convergence " outwards.	4 ma.	7°17′	124.5 7°6′	60.5 6°54′	39.1 6°42′	28.5 6°30′	22.1 6°18′	17.8 6°6′	14.7 5°54′	12.5 5°41′	10.7 5°30′	9.3 5°18′	7.1 4°54′	5.6 4°30′	4.5 4°6′	3.6 3°4[illegible]′	2.9 3°18′
		3 ma.	5°29′	93.3 5°20′	45.3 5°11′	29.3 5°2′	21.3 4°53′	16.5 4°44′	13.3 4°35′	11.1 4°26′	9.3 4°17′	8.0 4°7′	6.9 3°59′	5.3 3°41′	4.2 3°23′	3.3 3°5′	2.7 2°47′	2.1 2°29′
		2 ma.	3°40′	62.2 3°33′	30.2 3°28′	19.6 3°21′	14.2 3°16′	11.0 3°9′	8.9 3°4′	7.4 2°57′	6.2 2°52′	5.3 3°45′	4.6 2°40′	3.6 2°27′	2.8 2°15′	2.2 2°3′	1.8 1°51′	1.4 1°39′
		1 ma.	1°50′	31.12 1°47′	15.121 1°44′	9.787 1°41′	7.121 1°38′	5.521 1°35′	4.454 1°32′	3.692 1°29′	3.121 1°26′	2.676 1°23′	2.321 1°20′	1.787 1°14′	1.406 1°8′	1.121 1°2′	.898 55′	.721 50′
			1° 0	16.975 58′	8.248 57′	5.339 55′	3.884 53′	3.011 52′	2.429 50′	2.014 48′	1.702 47′	1.460 45′	1.266 43′	.975 40′	.767 37′	.611 33′	.490 30′	.393 27′
CONCAVE.	Convergence decentration inwards. Divergence " outwards.		1°	17.935 1°2′	9.207 1°3′	6.298 1°5′	4.843 1°6′	3.970 1°8′	3.389 1°10′	2.973 1°12′	2.661 1°13′	2.419 1°15′	2.225 1°16′	1.934 1°20′	1.726 1°23′	1.570 1°26′	1.449 1°30′	1.352 1°33′
		1 ma.	1°50′	32.879 1°53′	16.879 1°56′	11.546 1°59′	8.879 2°2′	7.279 2°5′	6.212 2°8′	5.450 2°11′	4.879 2°14′	4.435 2°17′	4.079 2°20′	3.546 2°26′	3.165 2°32′	2.879 2°38′	2.657 2°44′	2.479 2°50′
		2 ma.	3°40′	65.75 3°45′	33.7 3°52′	23.1 3°57′	17.7 4°4′	14.5 4°10′	12.4 4°15′	10.9 4°22′	9.7 4°28′	8.8 4°34′	8.1 4°40′	7.1 4°52′	6.3 5°4′	5.7 5°16′	5.3 5°28′	4.9 5°36′
		3 ma.	5°29′	98.6 5°38′	50.6 5°47′	34.6 5°56′	26.6 6°5′	21.8 6°14′	18.6 6°23′	16.3 6°32′	14.6 6°41′	13.3 5°50′	12.2 6°59′	10.6 7°16′	9.5 7°24′	8.6 7°42′	7.9 8°10′	7.4 8°27′
		4 ma.	7°17′	131.5 7°29′	67.5 7°41′	46.2 7°53′	35.5 8°5′	29.1 8°17′	24.8 8°29′	21.8 8°41′	19.5 8°52′	17.7 9°4′	16.3 9°16′	14.2 9°39′	12.6 10°3′	11.5 10°26′	10.6 10°50′	9.9 11°12′

The object of observation is presumed to be at a distance of more than 6 metres from the patient.

The figures in larger type indicate the amount of decentration in millimetres.

The figures in smaller type represent the deviating power of the prisms whose action is equivalent to that of the decentration of the lenses.

DR. PERCIVAL'S TABLES FOR NEAR VISION.

TABLE VII.—(CONVEX.)

			1 D	2 D	3 D	4 D	5 D	6 D	7 D	8 D	9 D	10 D	12 D	14 D	16 D	18 D	20 D
DIVERGING.	4 ma.	7°17′	240.6 13°32′	118.5 13°20′	77.8 13°8′	57.5 12°57′	45.3 12°46′	37.2 12°34′	31.3 12°22′	27.0 12°11′	23.6 11°59′	20.9 11°48′	16.8 11°26′	13.9 11°2′	11.7 10°47′	10.0 10°21′	8.7 9°51′
	3 ma.	5°29′	206.6 11°40′	101.9 11°31′	67.1 11°23′	49.7 11°14′	39.2 11°6′	32.2 10°57′	27.2 10°48′	23.5 10°39′	20.6 10°31′	18.3 10°22′	14.8 10°4′	12.3 9°47′	10.4 9°29′	9.0 9°11′	7.8 8°54′
	2 ma.	3°40′	172.6 9°47′	85.4 9°42′	56.3 9°36′	41.8 9°30′	33.1 9°24′	27.3 9°18′	23.1 9°12′	20.0 9°7′	17.6 9°1′	15.7 88°55′	12.8 8°43′	10.7 8°31′	9.1 8°17′	7.9 8°7′	6.9 7°56′
	1 ma.	1°50′	138.6 7°53′	68.8 7°50′	45.6 7°48′	34.0 7°45′	27.0 7°42′	22.3 7°39′	19.0 7°36′	16.5 7°33′	14.6 7°30′	13.1 7°27′	10.7 7°21′	9.1 7°15′	7.8 7°9′	6.9 7°3′	6.1 6°57′
		0	104.6236 5°58′	52.3118 5°58′	34.8745 5°58′	26.1559 5°58′	20.9247 5°58′	17.4372 5°58′	14.9462 5°58′	13.0779 5°58′	11.6248 5°58′	10.4623 5°58′	8.7186 5°58′	7.4731 5°58′	6.5389 5°58′	5.8124 5°58′	5.2312 5°58′
CONVERGING.	1 ma.	1°50′	70.6 4°2′	35.7 4°5′	24.1 4°8′	18.3 4°11′	14.8 4°14′	12.5 4°18′	10.8 4°21′	9.6 4°23′	8.6 4°26′	7.8 4°29′	6.7 4°36′	5.8 4°41′	5.2 4°47′	4.7 4°53′	4.3 4°59′
	2 ma.	3°40′	36.6 2°6′	19.2 2°12′	13.4 2°18′	10.5 2°24′	8.7 2°30′	7.6 2°36′	6.7 2°42′	6.1 2°48′	5.6 2°54′	5.2 3°	4.6 3°12′	4.2 3°24′	3.9 3°36′	3.7 3°48′	3.5 4°
	3 ma.	5°29′	2.6 9′	2.6 18′	2.6 27′	2.6 36′	2.6 45′	2.6 54′	2.6 1°3′	2.6 1°13′	2.6 1°22′	2.6 1°31′	2.6 1°49′	2.6 2°7′	2.6 2°25′	2.6 2°43′	2.6 3°1′
	4 ma.	7°17′	—31.3 (1°48′)	—13.9 (1°36′)	—8.1 (1°24′)	—5.2 (1°12′)	—3.4 (59′)	—2.3 (47′)	—1.4 (35′)	—.84 (23′)	—.36 (11′)	.03 1′	.61 25′	1.02 49′	1.3 1°13′	1.58 1°37′	1.77 2°2′
Difference for 1 ma.			33.995 1°57′	16.558 1°54′	10.743 1°51′	7.839 1°48′	6.0957 1°45′	4.933 1°42′	4.1029 1°39′	3.4801 1°36′	2.9957 1°33′	2.6082 1°30′	2.0270 1°24′	1.6119 1°18′	1.3004 1°12′	1.0582 1°6′	.8645 59′
Difference for 1°			18.54 1°3′	9.032 1°2′	5.861 1°	4.276 59′	3.325 57′	2.691 55′	2.2380 53′	1.8983 52′	1.6341 50′	1.4227 49′	1.1057 46′	.8792 42′	7.093 39′	.5772 36′	.471 32′

The object of observation is presumed to be $\frac{1}{3}$ metre from the centre of rotation of the globe.

The figures in larger type give the amount of decentration in millimetres.

—^{ve} sign indicates decentration outwards; +^{ve} sign decentration inwards.

The figures in smaller type represent the deviating power of the prisms whose action is equivalent to that of the decentration of the lenses. When enclosed in brackets the prisms are adducting in function, and should be placed edges inwards.

[P.T.O.

DR. PERCIVAL'S TABLES FOR NEAR VISION.

TABLE VIII.—(CONCAVE.)

			—1 D	—2 D	—3 D	—4 D	—5 D	—6 D	—7 D	—8 D	—9 D	10 D	—12 D	—14 D	—16 D	—18 D	—20 D
DIVERGING.	4 ma.	7°17′	—247.6 13°55′	—125.6 14°6′	—84.9 14°17′	—64.5 14°28′	—52.3 14°40′	—44.2 14°51′	—38.4 15°2′	—34.0 15°14′	—30.6 15°25′	—27.9 15°36′	—23.8 15°59′	—20.9 16°21′	—18.8 16°43′	—17.1 17°5′	—15.7 17°27′
	3 ma.	5°29′	—211.9 11°58′	—107.2 12°7′	—72.4 12°15′	—54.9 12°24′	—44.5 12°32′	—37.5 12°41′	—32.5 12°49′	—28.8 12°58′	—25.9 13°7′	—23.5 13°16′	—20.1 13°33′	—17.6 13°50′	—15.7 14°7′	—14.3 14°24′	—13.1 14°41′
	2 ma.	3°40′	—176.1 9°59′	—88.9 10°5′	—59.9 10°11′	—45.3 10°17′	—36.6 10°23′	—30.8 10°29′	—26.7 10°35′	—23.5 10°40′	—21.1 10°46′	—19.2 10°52′	—16.3 11°4′	—14.2 11°15′	—12.6 11°27′	—11.4 11°39′	—10.5 11°50′
	1 ma.	1°50′	—140.4 7°59′	—70.6 8°2′	—47.4 8°5′	—35.7 8°8′	—28.8 8°11′	—24.1 8°14′	—20.8 8°17′	—18.3 8°20′	—16.4 8°23′	—14.8 8°26′	—12.5 8°32′	—10.8 8°38′	—9.6 8°44′	—8.6 8°50′	—7.8 8°56′
		0	--104.6236 5°58′	--52.3118 5°58′	--34.8745 5°58′	--26.1559 5°58′	--20.9247 5°58′	--17.4372 5°58′	--14.9462 5°58′	--13.0779 5°58′	--11.6248 5°58	--10.4623 5°58′	--8.7186 5°58′	--7.4731 5°58′	--6.539 5°58′	--5.8124 5°58′	--5.2312 5°58′
CONVERGING.	1 ma.	1°50′	—68.9 3°56′	—34.0 3°53′	—22.4 3°50′	—16.5 3°47′	—13.1 3°44′	—10.7 3°41′	—9.1 3°38′	—7.8 3°35′	—6.9 3°32′	—6.1 3°29′	—4.9 3°23′	—4.1 3°17′	—3.5 3°11′	—3.0 3°5′	—2.6 2°59′
	2 ma.	3°40′	—33.1 1°54′	—15.7 1°48′	—9.8 1°42′	—6.9 1°36′	—5.2 1°30′	—4.0 1°24′	—3.2 1°18′	—2.6 1°12′	—2.1 1°5′	—1.7 59′	—1.1 47′	—.73 35′	—.42 23′	—.18 11′	—.01 (1′)
	3 ma.	5°29′	2.6 (9′)	2.6 (18′)	2.6 (27′)	+2.6 (36′)	2.6 (45′)	2.6 (54′)	2.6 (1°3′)	2.6 (1°12′)	2.6 (1°21′)	2.6 (1°31′)	2.6 (1°49′)	2.6 (2°7′)	2.6 (2°25′)	2.6 (2°43′)	2.6 (3°1′)
	4 ma.	7°17′	38.4 (2°12′)	20.9 (2°24′)	15.1 (2°36′)	12.2 (2°48′)	10.5 (3°)	9.3 (3°12′)	8.5 (3°24′)	7.9 (3°36′)	7.4 (3°48′)	7.0 (4°)	6.4 (4°24′)	6.0 (4°48′)	5.7 (5°12′)	5.4 (5°36′)	5.2 (°)
Difference for 1 ma.			35.754 2°3′	18.316 2°6′	12.504 2°9′	9.598 2°12′	7.854 2°15′	6.6916 2°18′	5.8613 2°21′	5.2385 2°24′	4.7541 2°27′	4.3667 2°30′	3.7854 2°36′	3.3702 2°42′	3.0588 2°48′	2.8167 2°54′	2.6229 3°
Difference for 1°			19.502 1°7′	10.088 1°9′	6.820 1°10′	5.235 1°12′	4.284 1°14′	3.650 1°15′	3.1971 1°17′	2.8574 1°18′	2.5932 1°20′	2.3819 1°22′	2.0648 1°25′	1.8383 1°28′	1.6685 1°32′	1.5364 1°35′	1.4307 1°38′

The object of observation is presumed to be $\frac{1}{3}$ metre from the centre of rotation of the globe.

The figures in larger type give the amount of decentration in millimetres.

—^{ve} sign indicates decentration outwards; +^{ve} sign decentration inwards.

The figures in smaller type represent the deviating power of the prisms whose action is equivalent to that of the decentration of the lenses. When enclosed in brackets the prisms are abducting in function, and should be placed edges inwards.

combined with a prism of 1° 24′ *d*, which is practically the same thing. Table VII. and Table VIII. are useful also in estimating the relative range of convergence for reading distance. A myope using − 5 D for reading can obtain binocular vision with a 3° prism held edge inwards before each eye, as well as with a 12° 32′ prism held edge outwards. His relative range is not, however, represented by 15° 32′, but by 12° 46′, or 7 ma.

CHAPTER XIII.

THE CLINICAL APPLICATIONS OF PRISMS IN DIAGNOSIS.

1.—To measure *the functional* minimum of convergence* (r^c). This is generally a negative quantity, so that the minimum of convergence of the visual axes is synonymous with their maximum of divergence. It is, in other words, the *maximum binocular abduction* of the visual axes in distant vision. Its measure is therefore taken by finding the strongest prisms with their edge outwards compatible with single vision of some distant object, as shewn in *Fig.* 54. The *deviating* angle of each prism ex-

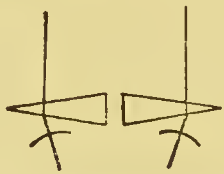

Fig. 54.—The functional minimum of convergence.

presses the "functional minimum of convergence"

* The word "functional" is used in contrast with the "absolute" minimum, which is the anatomical position of the eyes.

for each eye. The figure also makes clear that the visual axes meet behind the head during the experiment, and their point of intersection is "the far point of convergence." If a single prism be used, since its effect is distributed between the two eyes, its angle of deviation must be mentally halved for each. Remoteness of the point of fixation during the experiment is an important consideration if it is desired to reach the real minimum, for the knowledge of the object being at a great distance favours the relaxation of convergence. In ametropia, if lenses of sufficient strength are placed in a trial-frame, the experiment can be made by separating them if concave, or approximating them if convex, till double vision occurs, and then the prismatic effect can be calculated from rule I, p. 110, or the Tables by Dr. Percival.

Thus, if the distance between the optical centres of lenses of -4D be increased till it is 16 mm. greater than the distance between the centres of motion of the eyes, we divide the 16 mm. between the two eyes, for clearly each lens is displaced 8 mm., and therefore there is, for each eye, $1° 40'$ of divergence. It is expressed by saying that the functional minimum of convergence is $-1° 40'$. Dr. Risley's Rotary Prism, p. 48, or the apparatus in *Fig.* 25, affords the easiest and best means of making this test.

With vision for six metres, the normal power of

abduction is 2° for each eye, and is therefore measured by a single prism of 8°. The average abduction is greater in myopia and less in emmetropia, according to Schuurman. In testing the power of abduction with *M.*, refraction should first be corrected, and the effect of the lenses on the direction of the visual lines be allowed for. In *H.*, the test should be made both with and without lenses.

2.—To measure the *absolute maximum of convergence* (p^c). Find the strongest adducting prisms, as in *Fig.* 55, compatible with single vision of a

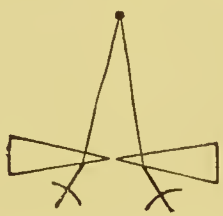

Fig. 55.—The absolute maximum of convergence.

test type or printed page held as close to the eyes as accommodation will permit. A strong prism can be placed in a trial frame before one eye, and Dr. Risley's Rotary Prism before the other, or the "prismatic trial frame" of *Fig.* 25 can be used, which possesses the advantage of dividing equally between the eyes the prismatic astigmatism and chromatic aberration.

This test, though described for completeness, is not easily enough made with ordinary prisms to

be much used, or to replace in practice the old established custom of advancing a finger towards the root of the nose till one eye deviates outwards, or the use of Dr. Landolt's Dynamometer in which the finger is replaced by a line of light. The two last tests, however, it may be suggested, do not give the "*absolute* maximum of convergence," for as soon as the point of fixation passes within the binocular near point of accommodation, diffusion circles appear on the retinæ, and increase rapidly in size with every approach till the eyes resign the impossible task of accommodation—then the ciliary muscle relaxes, and with it the convergence.

These tests depend largely, I believe, on the point of *the resignation of accommodative effort.* Even were the accommodative efforts to persist, it is scarcely to be expected that two pictures on their respective retinæ, each composed of a mass of diffusion circles, should excite the desire for fusion in its full strength, as clearly defined pictures would. At first sight the highest adducting prisms compatible with single vision of a test type held at the absolute near point of accommodation *would* seem to give the true "absolute maximum" of convergence; but here there is a new difficulty, for the chromatic dispersion of the prisms spoils the definition of the pictures on the retinæ, though truly it might be avoided by using a monochromatic

light.* By adding the maximum convergence to the already ascertained "minimum convergence," we get the "absolute range (amplitude) of convergence" (a^c), of which, according to Dr. Landolt, not more than one third or one fourth can be continuously in exercise for comfortable vision. I have never, however, worked by this rule, as Dr. Landolt does.

3.—To measure the "*relative range of convergence*" (a_1^c). For this, perhaps more valuable

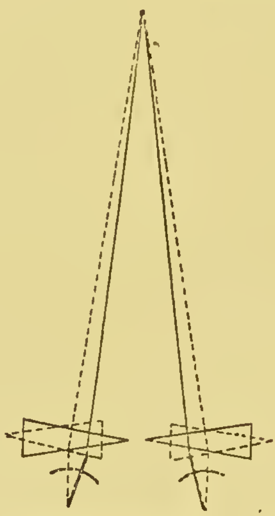

Fig. 56.—The relative range of convergence.

experiment, we find the strongest pair of abducting prisms, as shewn in dotted outline in *Fig.* 56, and

* Such light may be obtained of a pure yellow by placing common salt, or of a pure red by placing some salt of strontium upon the wick of a spirit lamp, of which the flame is kept steady by incombustible gauze, but the illumination is far too feeble to be of any use, and I do not in practice recommend the test at all.

then the strongest pair of adducting prisms, as shewn in continuous outline, compatible with single distinct vision of an object or type at some chosen distance. The distance most suitable is that at which the patient's daily work is done (occupation distance), and for which spectacles are most required.

The abducting prisms measure the negative part of the range (r_1^c), and the adducting prisms its positive part (p_1^c). This test, though beset with practical difficulties in its application, would be the most scientific of any, if the best ratio between the negative and positive parts were well worked out for different distances of vision.

The following table shews the relative convergence in a man of 32, with conditions otherwise

TABLE IX.

	A. Single Prism.	*D*. Deflection of *each* axis.	*R*. Relative Range.
6 m.	$+16°:-3°$	$+4\frac{1}{2}°:-\frac{3}{4}°$	$5\frac{1}{4}°$
1 m.	$+18°:-8°$	$+5°:-2°$	$7°$
$\frac{1}{2}$ m.	$+20°:-12°$	$+5\frac{1}{2}°:-3\frac{1}{4}°$	$8\frac{3}{4}°$
$\frac{1}{3}$ m.	$+24°:-16°$	$+6\frac{3}{4}°:-4\frac{1}{2}°$	$11\frac{1}{4}°$
$\frac{1}{4}$ m.	$+30°:-18°$	$+9°:-5°$	$14°$

normal. The first column *A* gives the refracting angle of the highest + and − prism he can overcome at the various distances. The second column *D* shews the deviation of *each* eye produced by the

aforesaid prisms; it is found by calculating the deviating angle of each prism, and dividing it between the two eyes. Thus the deviating angle of a prism of 16° is 9° *d*, and half this is 4½°, so that at 6 m. each eye can converge 4½° without losing the distinctness of an object. The third column *R* gives the relative range for each eye, calculated from the figures in column *D*.

It will be noticed that both the positive and negative parts of the range increase as vision becomes nearer; also that the positive part exceeds the negative, though less so as vision becomes nearer. The case cannot be taken as expressing the normal amounts, but the features just mentioned will, I think, be found to prevail. The table agrees more closely with Nagel's chart than with Donders'. The results obtained vary not only according to the peculiarities of the individual, but according to the method pursued. Thus, if we test the negative side of the range first, the positive side will be smaller than if we began with it first. As a rule, the limit of the negative part shews itself by the appearance of diplopia, and of the positive part by indistinctness from commencing excess of accommodation. The impairment of distinctness by strong prisms is apt to be confounded with this latter, so it is not easy to get exact results. On the whole, though I once attached considerable importance to the relative

range, and, indeed, it is theoretically important, I do not now commend it in practice except for a few rare cases requiring more careful investigation than we are generally able to give. The best apparatus for testing would be that of *Fig.* 25.

4.—*To dissociate convergence and accommodation.* Von Graefe was the first to suggest the use of prisms for this purpose. By placing a vertical prism before one eye, strong enough to create vertical diplopia, the reflex stimulus to fusion is suppressed, and the eyes assume what V. Graefe called their "position of equilibrium." In distant vision this position is, in the case of an emmetrope, that of the tonic convergence, but in near vision there is the accommodative convergence as well. A prism of four degrees ($2°d$) is generally strong enough to create vertical diplopia which cannot be overcome in distant vision, but for near vision a stronger prism must be used (*cf.* p. 69). V. Graefe used one of 15°. The distant object which he employed was a candle flame, the false image of which lay in the same vertical line when the equilibrium was perfect, while homonymous or heteronymous diplopia indicated respectively relative convergence or divergence. His object for near vision was a dot on a piece of card with a vertical line through it, so that the prism reduplicated the dot, but only lengthened the line. The disadvantage of this device was that

the overlapping portions of the line were sufficient to maintain the exercise of fusion, and thus mask any tendency to latent deviation that might exist. It was only considerable insufficiencies that manifested themselves by this test. The *degree* of deviation was measured by finding what additional horizontal prism was required to bring the candle flames of the distant test into one vertical line, and to unite the dots and lines of the near test; or, without using a second prism, the first prism may be rotated till the images are vertical, and then from the strength of the prism, and the amount of rotation, a calculation can be made of the deviation of the eyes.*

Since V. Graefe's near and distant tests both require a simple vertical prism, they give a false result unless the prism is held exactly vertical, and the head held perfectly steady. The slightest obliquity of the prism introduces a lateral displacement of the false image, which we may easily mistake for a

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* The formula is as follows, where d is the deviating angle of the prism, r its angle of rotation, and H the angle by which the eye deviates, and by which, therefore, the false image is misplaced,

$$\text{Tan. H} = \text{Tan. } d \text{ Sin. } r.$$

In words, the tangent of the horizontal angle we wish to measure, through which the false image is misplaced, is the multiple of the sine of the angle through which the prism has been rotated, and the tangent of the angle by which it deflects rays of light.

latent deviation where none such exists. To remedy this I made use, several years ago, of a double prism, which is shewn in *Fig.* 57. It is composed of two prisms, each of 2° or 3° united by their

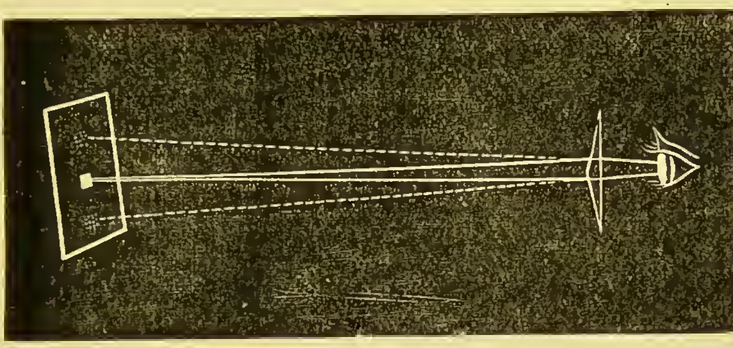

Fig. 57.—A double prism.

bases. The patient, shutting the left eye, holds this prism before the right one and looks through it at a flame, two images of which are seen, one above and one below the real position of the flame. Nothing is easier than to place the prism so that the two images appear vertical. On opening the left eye the flame is seen mid-way between its two images, and in the same vertical line if there be no latent deviation.* The double prism makes the test more delicate than a single one, but it is now replaced by the still better plan of a glass rod, suggested by a faint streak of light seen uniting the

* When I devised the double prism, I was not aware that a similar one, called a bi-prism, had been used by Fresnel to demonstrate the phenomena of "interference."

two false images, due to the fact, proved by Mr. Berry, that the edge of the obtuse prism is not a mathematical line, but from imperfect manufacture, a rounded ridge. It was easy to deduce from this that a glass rod would produce a better streak of light, by acting as a strong cylindrical lens.*

The first form of this little instrument was that shewn in *Fig.* 58, consisting of a short glass rod,

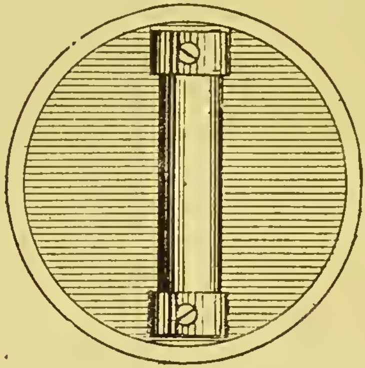

Fig. 58.—The first form of the glass rod test.

mounted in a metal disc. A convenient modification consists of a series of glass rods, placed parallel and in apposition, and united at their ends† by sealing wax. This form anyone can make for himself by breaking up a thin glass rod into several pieces an inch long, laying them together on a

* "Oph. Review," May, 1890.

† For a figure of this form, suggested by Mr. Berry's proposal to use corrugated glass, the reader is referred to Mr. Berry's "Diseases of the Eye," second edition, *Fig.* 166 *B.*

smooth level surface, and sealing their ends together. On looking at a distant flame with this before one eye, it appears converted into a long streak of light, which there is no desire to regard as a false image of the flame, from its dissimilarity, especially if red glass be used. If the streak pass through the flame, equilibrium is perfect, but if otherwise, its distance indicates the amount of latent deviation. The prism that is able to bring the line and the flame together is the measure of it. A very convenient way, in the absence of a tangent scale (*Fig.* 59), is to place the rod test in one side of a trial frame, and Risley's "rotary prism" in the other, and turn the latter till the line crosses the flame. Scales graduated in degrees, as in *Fig.* 59,

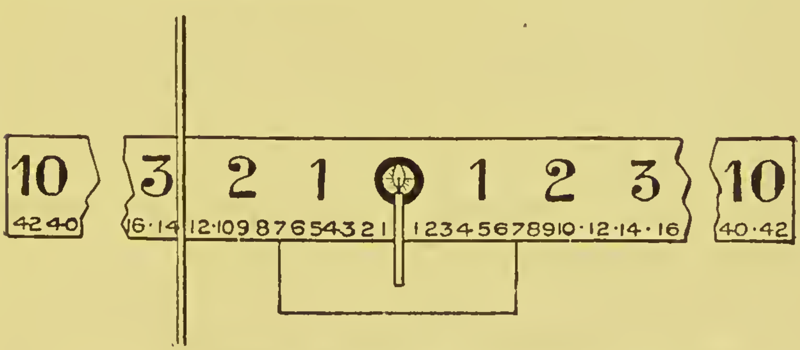

Fig. 59.—Tangent scale for use with the glass rod test.

afford the best means, however, since the patient can read at any moment the figure crossed by the line of light, and thus variations can be followed. The full extent of a latent deviation does not occur at once, but the line gradually moves farther

from the flame. Such a scale, graduated for use at 5 metres, and with directions for use, can be obtained from Messrs. Curry and Paxton: it is also furnished with smaller figures for the objective measurement of squint, but that does not belong to the subject before us.

For near vision, a tangent scale is published, graduated in degrees and metre-angles, for ¼ metre, with a vertical arrow running up from zero, such that when reduplicated, as in *Fig.* 43, by a square vertical prism held before one eye, the lower arrow points to that figure on the upper scale which measures the deviation. It is best to use a prism of 12° (6° *d*), permanently mounted in an ordinary strong cataract frame, which can be placed on the patient in a moment, and needs no hand to hold it. A thread, ¼ metre long, should be permanently fixed to the card to adjust its distance from the patient correctly. This scale can also be obtained from Messrs. Curry and Paxton, provided with a printed sentence, as well as the figures, to ensure full accommodation—an improvement suggested by Mr. Berry. The rod test can be used instead of a prism if the arrow be replaced by a small polished metal button to reflect the light from a window or a flame. The illumination, however, is rather too feeble to be of much use clinically. The best device for economising the light, by admitting the maximum amount into the pupil, is a convex cylindrical

lens, with a radius of curvature of about 20 mm. By holding the scale vertically, instead of horizontally, we can, by this method, measure latent vertical deviations, for the estimation of which, in near vision, we possess no other very satisfactory method.

Tangent scales can be made for any intermediate distance, but though I have used them for physiological investigation, they are not necessary in practice. If graduated in degrees or metre angles, a different scale is of course needed for each distance; while if marked in centimetres, and provided with an arrow capable of being lengthened or shortened, one is sufficient, but then a calculation is necessary.

What is the due significance to attach to the near vision test? It clearly does not decide the strength of one rectus, for convergence is a single function affecting both eyes equally. It is true that each rectus is a link in the chain of convergence, and *may* be the weak link, but that would shew itself best in testing the lateral movements of the eyes. Neither does it afford a pure test of the strength of convergence, for the amplitude of convergence may be quite normal, and yet an "insufficiency" (as it used to be called) appears in the test for latent equilibrium, from one of its stimuli being wanting, as in myopia, where accommodation is slight, or absent. The section on

"The study of convergence," contains an account of the conditions that need consideration.

5.—*To detect vertical latent deviations* (hyperphoria). These are of two kinds, paretic and concomitant. The name "simple hyperphoria" should be confined to the latter. If the deviation be of the same amount on looking upwards or downwards, it is concomitant; if otherwise, paretic.

The old way of disclosing such deviations was by placing before one eye a prism, with its edge outwards, and slightly stronger than can be overcome—say one of 6° *d*. In this test especially, precision in the setting of the prism is essential, since imperfect setting, or an inclination of the patient's head, cause apparent hyperphoria, when such does not exist, as shewn in *Fig.* 19. Slight vertical deviations are less frequent and more important than slight horizontal ones, since in all normal individuals the vertical equilibrium is perfect. The glass rod is a delicate test for it, free from the sources of error referred to in the use of a prism, since slight rotations of the rod, or of the head, do not appreciably affect the distance of the line of light from the flame. The object should be a gas-jet turned down to about a quarter of an inch in height, at the distance of 5 or 6 metres, while a piece of blue glass held before the other eye greatly improves the test, by subduing the illumination of the flame, which otherwise is apt to eclipse the line

by its superior brightness. Care should be taken that direct light from no other source, such as a window or a fire, falls on the glass rod, and this can be obtained by standing so as to place it in one's own shadow: a dark room is not necessary. The measure of the hyperphoria can be taken by finding what prism brings the flame and line together, or how much rotation must be given to Risley's "rotary prism." A scale of degrees, as shewn in *Fig.* 60, is, however, what I have generally used.

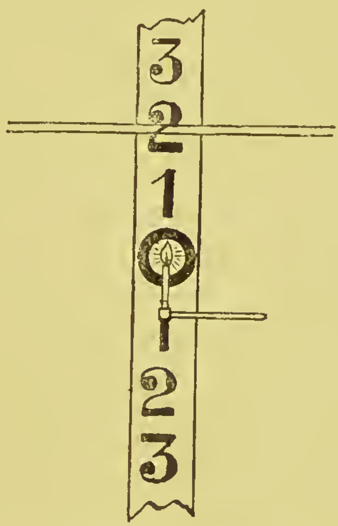

Fig. 60.—A tangent scale to measure hyperphoria with a glass rod.

For *near* vision it is not so easy to measure hyperphoria, nor is it often required. The plan mentioned on page 129 can be used, or a stereoscope, marked before one eye with a vertical line graduated in degrees up and down from a central zero

point, and before the other eye with a horizontal line exactly level with the zero point just mentioned. On looking into the instrument, the horizontal line appears to be across the zero of the vertical one if equilibrium is perfect, but if not, the figure crossed by the horizontal line measures the hyperphoria. Stevens* suggests the desirability of finding the highest prism, edge up, and the highest, edge down, compatible with single vision. Instances of hyperphoria are not so rare as they are thought to be. It is well, when opportunity offers, to test them again after a considerable interval to see if the amount is stationary: often their persistency is remarkable, and the same case will show year after year, exactly the same measurement. Even slight hyperphoriæ sometimes cause great inconvenience, and even headache and asthenopia. On the other hand, hyperphoria may be very considerable without causing a single symptom, or seeming to be of any consequence. I have recorded elsewhere† the case of a young man, 18 years of age, who had right hyperphoria of nearly 10°, without any headache or discomfort from it. His right eye was noticed by his mother to occasionally "roll up," and since his grandmother was said to exhibit the same phenomenon, it is more than likely that

* "Archives d' Ophthalmologie" XVI.; 6; p. 544.

† "Ophth. Review," Vol. XII., p. 44.

there was hereditary transmission of the defect, and its toleration would thus be accounted for. This case called for no treatment, since the anomaly caused no discomfort; tenotomy of the right superior rectus, or advancement of the left inferior, would have been indicated had distressing symptoms been present.

In true hyperphoria one eye turns up when it is covered, as much as the other turns down when it is covered. Mr. Berry has noticed some rare aberrant forms of hyperphoria, in which one eye turns up without the other turning down, or in which either eye turns up when covered. In paretic hyperphoria the latent deviation is greater in some directions than in others, and by using the rod test, with the patient's head in different positions, it is easy to discover which eye is at fault, and which muscle is affected.* For the correction of hyperphoria, see page 140.

6.—*Prisms are sometimes used to measure strabismus*, or the degree of any existing diplopia, by finding what prism is able to unite the two images, but this plan is fallacious since prisms of different strengths will be found equally to succeed. The weakest prism which enables the two images to be united, though it does not measure the diplopia, yet gives us the useful information of how much

* "Ophthalmic Review," Vol. IX., p. 287.

the diplopia needs to be lessened in order to allow the remainder to be corrected by the desire for single vision. The deviating angle of the prism tells us the proportion of diplopia which is beyond the control of fusion. It is sometimes recommended to prescribe prisms of this kind, so as to exercise the neuro-motor mechanism till the strength is regained.

Since homonymous diplopia is due to excessive convergence, we use *plus* prisms (*i.e.* edge in), and since heteronymous diplopia is due to excessive divergence, we use for it *minus* prisms (*i.e.* edge out). In simpler language, " place the apex of the prism in the same direction as the eye deviates " (Juler).

7.—*To decide the presence or absence of binocular vision.*

The fact that a prism before an eye deflects the line of fixation was turned to account by Von Graefe, in cases of suspected malingering, to see whether an eye *has such a thing* as a line of fixation, or in other words whether it is a seeing eye. The following is perhaps the fullest plan. After fixing the patient's attention, we may first place a strong prism, say of 30° (edge in or out), before the *sound* eye. If, while doing this, the other betray no associated movement, vision is almost certainly binocular, for nothing but fixation could preserve it from accompanying the deflection of its fellow,

except in paralytic cases. Should it move, however, nothing is proved beyond the fact that its fixation power is less perfect than that of the other.

The same prism is now similarly placed before the *doubtful* eye; if while doing so the other display an associated movement, vision is not only binocular, but the impugned eye is the better of the two, at least for the distance of vision employed at the time: but if there be no associated movement nothing is proved.

Lastly, on placing a plus prism of about 10° before the doubtful eye, and *quickly* withdrawing the prism, we closely watch the eye from which it is taken; for a corrective movement, if detected, at once tells us that its line of fixation has been deflected by the prism, and that it *has* therefore a line of fixation and is a seeing eye.

The subjective test, in which the patient's statements have to be relied on, is, for obvious reasons, often unsatisfactory, though it possesses the peculiar advantage of convincing a by-stander from the patient's own mouth, and of thus relieving the surgeon from the sole responsibility. It consists in placing a prism first before the unsound eye, and asking whether any object, preferably a flame, appears double. The reply is almost certainly negative. The prism is next placed before the sound eye, and the question repeated, "Do you

now?" If incautious, the patient may admit that though he sees nothing with the bad eye, he sees double when the prism is held before the good one.

To confuse the patient in a case suspected to be one of simulated unilateral blindness, Alfred Graefe suggested to show him, first of all, that diplopia is obtainable with one eye. This is best done by making him look at a distant flame with the impugned eye covered, while a square prism, the base of which is held uppermost and has sharp angles, is moved upwards till it reaches almost to the lower margin of the pupil. Double images of the flame are thus produced. On repeating the experiment, but this time without covering the impugned eye, the prism is held higher up so as to cover the whole pupil. If, under these conditions, diplopia results, it must be because both eyes see. It is much more difficult to produce monocular diplopia by placing the apex of a prism opposite the pupil, than by placing the base opposite its lower margin. Mr. Adams Frost has pointed out a possible source of fallacy in making this test, namely, that the image by double internal reflection, explained on page 51, may be noticed by the patient, who may thus say that he sees double, though the other eye be blind. The image spoken of is extremely faint, but though with a strong prism it is too far removed from the true image to attract attention, it might be noticed with a weak one.

8.—*To elicit diplopia in cases of suppression of the false image.* After a squint has lasted a sufficient length of time, the mind has so learned to ignore the false image that it is impossible to produce diplopia by looking at a flame. The false image is then said to be "mentally suppressed." If the suppression be not deep, attention can be called to the false image by placing a piece of green or red glass in front of the fixing eye, or, if this fail, by placing a vertical prism before either eye to throw the picture of the flame on to a different part of the retina. There are cases of even alternating squint in which the false image is suppressed, whichever eye fixes. The suppression is sometimes so deep, in long-standing cases, that nothing can overcome it. In a few cases the glass rod has elicited diplopia when prisms have failed to do so: it should be combined with a coloured glass before the other eye.

Chapter XIV.

THE USE OF PRISMS IN TREATMENT.

Prisms may be used,—

1.—*To relieve excess of tonic convergence.* We need first to exclude causes which can be relieved by constitutional treatment—reflex, hysterical, cerebral, etc. Hypermetropia needs correction. If the latent convergence does not exist for all distances of vision, care must be taken that the prisms ordered should only be worn for distant vision. If there be excess of convergence in near vision also, they can be worn for all purposes. In myopia it is not uncommon to find considerable latent convergence in distant vision giving rise to occasional diplopia, which can be remedied by displacing the lenses nearer together till the excess of convergence, shewn by the rod-test, is reduced almost to nil. It is well not to correct more than two-thirds of the latent deviation.* Slight excesses can be left alone.

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* Noyes recommends its full correction, on his supposition that it is due to insufficiency of the externi. This is not a supposition I feel able to endorse.

2.—*To relieve deficiency of tonic convergence.* This is a much commoner condition than the last, and can generally be left alone. It occasionally, however, gives rise to diplopia, or to asthenopia and headache. It may then be relieved by prisms which annul half the latent condition. Convex lenses can be displaced inwards, and concave lenses outwards. Latent divergence in distant vision is nearly always accompanied by still greater latent divergence in near vision, so that the same prisms can be worn for near work as for distant. In some neurasthenic individuals great variability occurs in the equilibrium of the eyes. These are not the cases for prisms.

3.—*To relieve excess of accommodative convergence.* It is extremely rarely that this occurs in other than hypermetropes, where correction of the refraction is all that is indicated. I have not met with an emmetropic case.

4.—*To relieve deficient accommodative convergence.* In myopia, since accommodation is less than in emmetropia, the accommodative convergence is less. This is rectified by wearing the full correction for the myopia, but sometimes the full correction cannot be borne, and it is advisable to prescribe weaker lenses either displaced or decentred outwards, or combined with prisms, edge out. What frequently happens is that both the tonic and accommodative convergences are deficient, and

then the correction of the former generally suffices if the same spectacles are used for all purposes. A high amount of latent divergence at reading distance is often tolerated without discomfort. It occurs too in cases other than myopic.

Illustrative case.—Mrs. L., 38 years of age, has had extensive corneal nebulæ from childhood. RV = $\frac{5}{60}$: LV = $\frac{5}{36}$. Equilibrium by rod test, and tangent scale,

$$5 \text{ m.} = -1^\circ$$
$$\tfrac{1}{4} \text{ m.} = -12^\circ$$

She is obliged to hold her sewing, or small print, near her eyes, to obtain sufficient visual acuity, yet this causes headache and asthenopia. As shewn above, she has slight latent divergence for distance, namely, 1°, which is $\frac{1}{2}^\circ$ for each eye, and can be neglected. At $\frac{1}{4}$ metre she has 12° of latent divergence, which is 6° for each eye. She was ordered, for fine work, spectacles of +3·5 D, with prisms of 6° *d*, base in. With these she can hold her work near, and get good acuity without discomfort; the nebulæ making her less observant of the prismatic aberrations. She could not tolerate them at the first trial, but in a day or two said, "I would not be without them," and writes after a few months: "spectacles very useful to me in reading."

5.—*To relieve hyperphoria.* This is one of the most useful of the services rendered by prisms. Well marked cases of persistent hyperphoria, with dis-

comfort from threatened diplopia, and eye strain, are immensely relieved by prisms, which should correct about two-thirds of the defect I think. It will be remembered that prisms set vertically have less effect over hyperphoria in near than in distant vision, p. 69, so that full correction for distance is under-correction for near. The prisms should be equally divided between the two eyes, the edge of each being in the same direction as the axis of the eye, that is edge up before the higher eye, and edge down before the lower.

Illustrative case.—Miss Mary T——, age about 45, with ·5 D of hypermetropia in each eye and V = $\frac{20}{20}$, complains that she has had a tendency to double vision for the last two years, and that when she does see double the objects appear superposed; thus she thought she saw two rows of cows, one above the other, discovering afterwards there was only one row. A similar difficulty, encountered in reading, she had learned to overcome by tilting her reading spectacles to make one lens higher than the other (an unconscious use of prisms). The rod test shews at 5 m. right hyperphoria (*i.e.* the right eye highest) of 1°. She also complains of headaches and asthenopia, such that, though highly intellectual, she has almost ceased to read or use her eyes for close work for the last year or two. She remembers that, when a child, she could not see properly through a stereoscope from the two images

appearing superposed. *Treatment.*—Prisms of $\frac{3}{4}°$ before each eye, edge up before right, edge down before left, with lenses of $+0·5$ D. Similar prisms to be combined with her reading spectacles. *Result.*—The headaches greatly relieved; asthenopia gone; says glasses have taken her back several years in the use of her eyes, so that she can sew and read again as of old, but if she takes them off for a moment the diplopia is more troublesome than before. This last fact illustrates a great disadvantage in the use of prisms prescribed for relief rather than cure. The prisms, being less than the hyperphoria, cannot really have created an increase, but have probably allowed more of what was latent to become manifest. This, however, is more than compensated for by the relief they have given.

6.—*To correct diplopia from oculo-motor paresis.* In recent cases we, of course, wait for cure of the condition, but if it seems to come to a standstill, we may afford relief by ordering prisms such that the images can be united in at least the most useful direction of vision. At first, the diplopia occurs only in a certain area of the field of fixation, but after a while through what is called "contracture of the antagonist" there is often latent diplopia over the whole field. This latent element should be corrected, and a little of the manifest as well. Prisms should have their edges placed in the direction in which the axis of the affected eye deviates,

remembering, if we wish to divide the prism between the two eyes, that the one before the sound eye should have its apex in exactly the opposite direction. If for instance the right eye deviate down and out, its prism should be apex "down and out," and the prism for the left eye be apex "up and out." In old paralysis of the superior oblique, prisms are very valuable, since on looking down at the work the diplopia is troublesome. The most thorough way to examine a case is to combine the vertical and horizontal scales of *Figs.* 59 and 60 with a flame at their centre, and then test the equilibrium of the eyes with the glass rod in each of the nine positions of the field of fixation; this is done by placing the patient's head in different positions. From a chart thus made, it is easy to reason out what prisms it is best to order. But a quicker way is just to place the patient's head so that on looking at the flame his axes of fixation are in the position mostly required at his work; the vertical and horizontal deviations can then be measured, and prisms be prescribed to correct a certain proportion of each. Thus, if the right eye deviate upwards 5° we may order a prism of 2° *d*, edge up, before the right eye, and the same, edge down, before the left. If the same eye also converges 3°, we may combine with each of the above a plus prism of 1° *d*, on the principle of "resultant prisms." A man with paralysis of the right

superior oblique, who could only look down at his work comfortably by shutting one eye to avoid diplopia, greatly valued resultant prisms of 3° ($1\frac{1}{2}°d$) each which were ordered him.

7.—*To assist in the cure of paralytic diplopia* (diphtheritic, rheumatic, etc.). Donders wrote, "We may further, in paresis of a muscle, so far meet the disease by means of a prism, that in order to make the double images which have been brought near one another run together, the muscles will become powerfully tense, which, for the alleviation of the paresis, appears to be no matter of indifference."* In recent cases, without secondary contraction of the antagonist, this plan is not very feasible. In older cases, where the defect is slight, it might possibly serve a purpose, but I doubt if it would really have a curative effect since the good it is supposed to do in one part of the field would be undone in another part, unless the field of diplopia were very extensive.

8.—*To disguise a squint in an amblyopic eye.* The cosmetic use of prisms was brought to my notice by a case in the practice of Mr. Berry, who succeeded in concealing, to a considerable degree, a vertical squint in a young woman, by prescribing a vertical prism to be worn before it, in conjunction with a lens equal to that needed before the seeing eye. Since prisms cause apparent displacement of

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* Anomalies, &c., p. 134.

objects towards their edges, the edge of a cosmetic prism should be in the opposite direction to the squint. Thus if the eye squint up, the edge of the prism should be down ; if the eye squint out, the edge should be in. This procedure is chiefly applicable to slight vertical squints, which are more disfiguring than slight horizontal ones.

Chapter XV.

ANALYSIS OF SPECTACLES.

Hitherto we have dealt with the synthetical part of our subject, how to prescribe prisms and combine them with lenses. The analytical part needs but brief consideration. When a patient comes to us with a pair of spectacles already in use, but which cause discomfort, in spite of having the correct focal length, how are we to investigate their prismatic effect, and find how the glasses are centred? Clinically, nothing is easier than to test the positions of equilibrium by the glass rod, etc., with and without the spectacles. We then see what effect they have on the visual axes, and by what distance lenses of similar focus, when placed in a trial-frame, need to be separated to produce a desirable equilibrium.

We can reach the same result in another way, by measuring the distance between the centres of the eyes and between the optical centres of the lenses. If these two distances are equal, and the spectacles be for distant vision, they have practically no effect on convergence. If, with reading spectacles, the distance between the

optical centres be 5 or 6 mm. less than the inter-ocular distance, the effect on convergence is also practically nil. In *Fig.* 61 is represented a rough

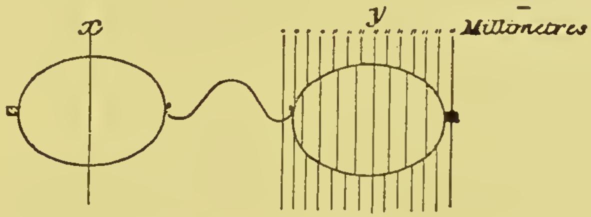

Fig. 61.—A simple way of telling the distance between the optical centres of a pair of spectacles.

but ready method of telling the distance between the optical centres. The spectacles are held horizontally, and some inches above a card marked to one side with a vertical line *x*, and towards the other side with parallel lines *y*, at different measured distances from the single one. The observer, holding his left eye about a foot above, and as nearly vertically over the left lens as he can judge, closes the right eye, and moves the spectacle frame from side to side till the line *x* appears unbroken by the left lens. Now, without moving the head, the right eye is opened and the left closed, and that line which appears to be least displaced by the right lens is the line nearest its optical centre, so that the number above it furnishes a rough estimate of the required distance between the two optical centres.

To ascertain whether each optical centre is

midway between the upper and lower border of its rim, each lens must be held lengthwise over the left hand line till it appears to run continuously. If we wish to measure the distance of each optical centre from the middle of the bridge, it is easily done by marking each lens with an ink dot where a single line (x in *Fig.* 61) appears unbroken, so as to measure the distance of each dot from the centre of the bridge. If the optical centre lie outside the lens altogether, then the "analyser" in *Fig.* 63, which was described in the first edition of this work, can be used, though it has since been made unnecessary by the introduction of the "prism measure" of the Geneva Optical Company. The analyser, however, though more difficult to use, possessed the advantage of giving a dioptric measurement true for media of any index of refraction, while the "prism measure" only gives the physical inclination of the surfaces. The latter instrument, shown in *Fig.* 62, is very simple in its use. If a lens be placed within it, and be moved about in the horizontal plane till the index points to zero, the tooth in the centre of the foot of the index points to the optical centre. If, on the other hand, the geometrical centre be placed just below this tooth, the index points to the number which measures the physical angle of the virtual prism with which the lens is combined. This does not in itself, however, give us much information about the

effect of the spectacles on the visual axes, for the geometrical centre may not be directly in front of the eye. If we measure from the centre of the bridge a distance equal to the distance of the centre of the eye from the median plane, and

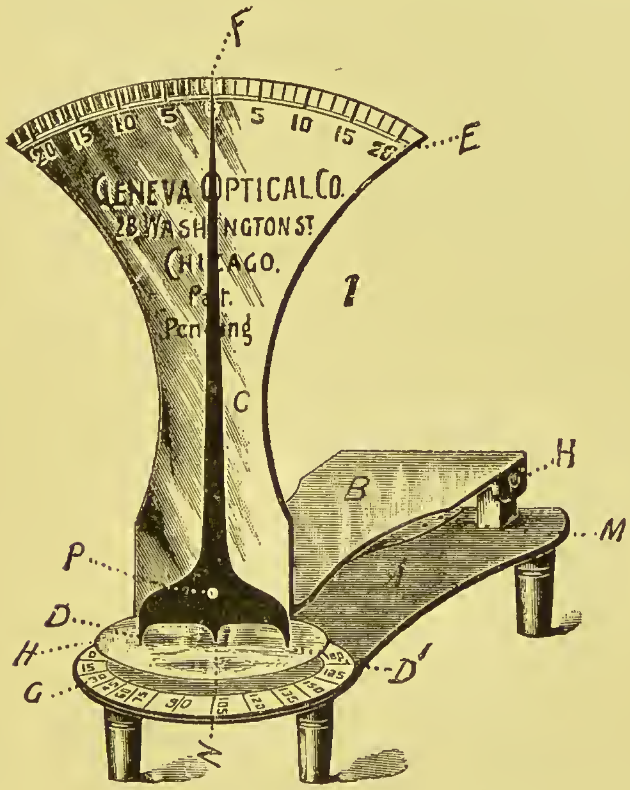

Fig. 62.—The "prism measure," used to discover the optical centre of a lens.

mark this distance by an ink dot on the glass, then we can place the said ink dot under the tooth and read off what prism is virtually combined with the lens at this point, directly in front of the eye. In this way we discover the prismatic effect of the

spectacles without any calculation. This application of the instrument, though not mentioned by its inventor, makes it as useful to the surgeon as it is to the optician, and renders any description of the analyser almost unnecessary, except for a few who may have had it constructed, or who wish great exactness with pebble lenses, as well as with glass ones.

The "analyser," which is shown in *Fig.* 63, has a plate of glass, *c d*, which is movable from side to side. The letters *a b* indicate a narrower plate of glass, also

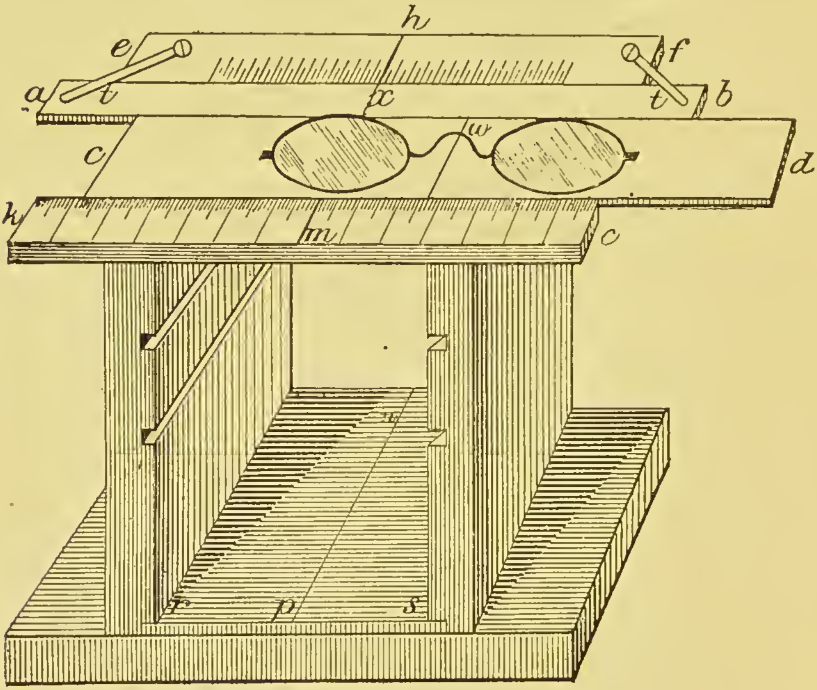

Fig. 63.—The Analyser.

movable, marked exactly across the middle with a transverse line *x*. On either side of the slides is

a millimetre scale with the zero in the centre. The broad slide has a long line *w* right across it opposite the zero of its scale. The *card r s* has only one line upon it, exactly beneath the central lines of the glass strips, and it can be placed either in the bottom of the apparatus, as in the figure, or in the grooves higher up, for lenses whose focal length is shorter than the height of the glass plates above the lowest grooves. The card is generally used in its lowest position, because distance of the line upon it from the lenses, as we have already seen with prisms, magnifies the phenomenon, and makes the experiment more delicate. The height of the upper surface of the glass plates from the card in the lowest grooves is about $4\frac{1}{2}$ inches.

To analyse a pair of spectacles, lay the centre of the bridge on the line *w*, and place the observing eye so that the lines *x* and *n p* appear to coincide. Now move the broad slide bearing the spectacles till the line *n p* appears undeviating through one lens; then *w* points to the distance between the optical centre of that lens and the centre of the bridge. Repeat in the same way for the other lens.

Lastly, by laying the geometrical centre of a lens opposite the zero of the scale *k e*, the line *n p* will, if it be a decentred lens, appear displaced through it. How great is the apparent displacement? By answering this we tell the prismatic equivalent of the decentering without recourse to any table. We must use another sight* fixed to the narrow slide perpendicularly beneath its line *x*, then by holding the eye so as to keep them both in

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* Another way is to use a plane mirror instead of the card *r s* and keep the line *x* in line with its reflection—a little difficult sometimes.

apparent coincidence (thus ensuring a vertical line of vision) and moving them till they appear exactly abreast of the apparent displacement of the line *n p* as seen through the lens, the prismatic equivalent of the decentering is indicated by the degree on the scale *e f* to which *x* points. The special applicability of this mode is when the optical centre lies outside the lens. When the lenses under test are of shorter focal distance than the height of the analyser, the card must be placed in one of the other two pairs of grooves: then, though the use of the scale *k e* is unchanged, it must be remembered that when the card is in the grooves half way up, each figure in the scale *e f* must be multiplied by two, and when the card is in the highest grooves, as for cataract lenses, each must be multiplied by four. The scale *e f* can be marked either in millimetres or in degrees; if the latter, then the height of the lenses above the card must be 53·3 mm. or some multiple of that number, for the degrees to be one millimetre from each other, or some multiple of a millimetre. But if marked in millimetres the height of the lenses above the card should be 100 mm., so that by a table of tangents affixed to the analyser the degree can be found. This is the most exact method, especially for strongly prismatic lenses

Chapter XVI.

LOCALISING THE EYES.

It is often desirable to measure the distance of the centre of each eye from the bridge of the nose, to estimate correctly any apparent want of symmetry, and to know at what distance from the middle line to place the geometrical or optical centre of a lens. Many tests exist for the mutual distance between the two eyes, of which, perhaps, the best is Snellen's frame, but they estimate the distance between the two pupils, whereas what we need for the exact study of prisms is the distance of the centre of rotation of each eye from the median plane which bisects the nose.

For a test to be true in principle, not only must the visual lines of the observing and observed eye coincide, but the line so composed must be exactly at right angles to the line uniting the centres of the two eyes, or, at least, if one eye be examined with any angle of inclination, the inclination must be the same in the examination of the other.* It need

* On the principle of the parallelogram.

hardly be said that this cannot be ensured when only a single point is used for a bearing. To overcome this difficulty a little device, *Fig.* 64, may

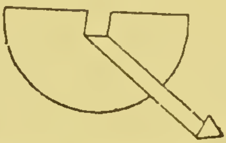

Fig. 64.—The Localiser.

be used with two sights so disposed as to always be in a line at right angles to any trial-frame in which it is placed. One pupil is localised at a time, all that is necessary being to increase or diminish the breadth of the frame till the two sights and the centre of the pupil are in line, as in *Fig.* 65; the patient meanwhile is bidden to be

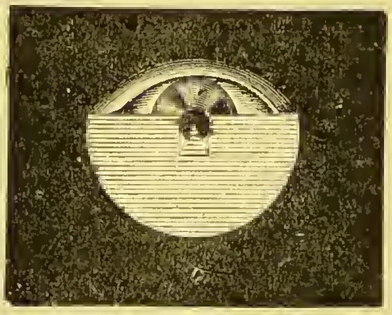

Fig. 65.—The Localiser in use.

looking at the pupil of the surgeon's observing eye, by doing which the two visual lines (*i.e.*, that of the observing and that of the observed eye) come to practically coincide. If the patient is a child it is quite easy to excite its interest in its own miniature

reflection in the surgeon's pupil while taking the observation, adeptness in which needs a few trials to become familiar with the parallactic motion. The instrument is then placed in the other rim of the frame, and the observation repeated for the other eye.

By this method we comply with the two afore-named conditions of accuracy; the visual lines of the observing and observed eyes coincide, and of necessity they are both at right angles to the trial-frame. The frame must, of course, be one movable by a screw, and marked in millimetres, so that the record can be read off after localising each pupil. The distance of the centre of each eye from the middle line is in each case *half* what is registered by the frame. It is true that the present methods may be considered sufficiently exact for practical purposes, but there is, nevertheless, a pleasure in working as truly as possible, and the frequent and careful use of this instrument will shew how many an unsuspected difference exists in the position of the two eyes. When ordinary spectacles are prescribed for eyes placed unsymmetrically, the prismatic effect is greater for one than the other, which, however, is only of importance with very strong lenses, being counteracted by involuntary inclination of the head; another consequence is that the fields of fixation for each eye are unsymmetrical.

TRIAL-FRAME FOR TEMPORARY PRISMS.

In the first edition of this work it was said that prisms should not be prescribed without wearing a temporary pair for a few evenings first, to learn how they suit, but I now think this rarely needful, for if the conditions of convergence be carefully studied we need hardly ever prescribe prisms without feeling pretty sure beforehand of their effects, even though, as is often equally the case with cylinders, they do not seem well borne just at first. Still, there are occasions where temporary prisms are of service, and the diagram of the frame to hold them may be reproduced, *Fig.* 66. It is made to hold

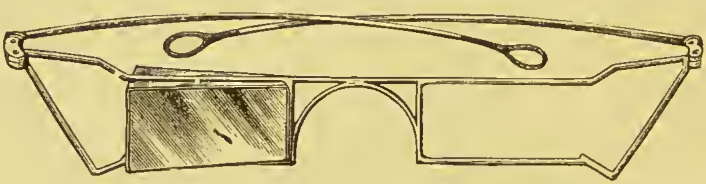

Fig. 66.—Rectangular prisms in a frame as suggested in the text. This frame may be obtained from Messrs. Curry & Paxton.

rectangular* prisms, the shape of which permits them to be adjusted at once, and removes any possibility of their displacement. Each prism has a longitudinal groove in its upper and lower surface, so that it need only be pushed in between the wires till caught by the elbow. So light a frame as this could be worn together with an ordinary pair

* Dr. Percival has proposed a slight modification, namely, that the prisms should be square instead of oblong.

of spectacles when it is desired to try for a few evenings the joint effect of prismatic and spherical glasses.*

I would recommend, in prescribing prisms, the following simple maxims :

1.—Never order strong prisms unless the indications for their use are unmistakable.

2.—Rarely order them stronger than 4° (2°*d*) unless there be amblyopia, as from corneal nebulæ, etc.

3.—Never give them on account of a mere anomaly in the behaviour of the eyes under the various tests, if it causes no fatigue or discomfort.

4.—Do not judge by the rod test alone, but also try the "finger test" of p. 119, and the "card test" of p. 128, and, if specially indicated, the "relative convergence test," p. 120.

5.—Remember that latent deviation in distant vision is more important than in near vision, though the latter is not to be ignored; and that moderate divergence in near vision is physiological.

6.—Always, more or less, under-correct.

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* It may be obtained from Messrs. Curry & Paxton.

APPENDIX.

1.—*Resultant Prisms.* For those who dislike calculations, the following method of combining a vertical with a horizontal prism may be of service. Draw a horizontal line as many inches long as there are degrees in the deviating angle of the horizontal prism. From one end of this line draw another vertical line as many inches long as there are degrees in the deviating angle of the vertical prism. Then the number of inches between the free ends of these two lines equals (approximately) the number of degrees in the required resultant prism; and the inclination of the base-apex line of the resultant prism to the horizontal is given by the angle between the horizontal line and the hypothenuse, or resultant line, which unites the free ends already spoken of.

2.—*Sines, Arcs, and Tangents. Figure* 67, to shew the relation between the arc, sine, and tangent, is introduced for the sake of any reader who may not be acquainted with trigonometrical ratios. Let any angle be included between the two radii of a circle, then the portion of the circumference included between the radii, as $a R$ in the figure, is

the *arc* of the angle, and, if the radius be taken as unity, a perpendicular (RS) dropped from the extremity of one radius perpendicularly upon the

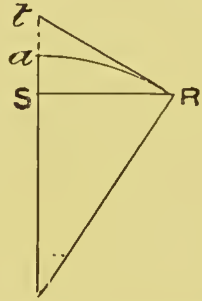

Fig. 67.—To illustrate the sine, arc, and tangent of an angle.

other radius is the *sine*, and a line drawn perpendicularly from the extremity of one radius to meet the other, as $R\,t$ in the figure, is the *tangent* of the angle. It will be seen at once that the *sine* is always less, and the *tangent* always greater, than

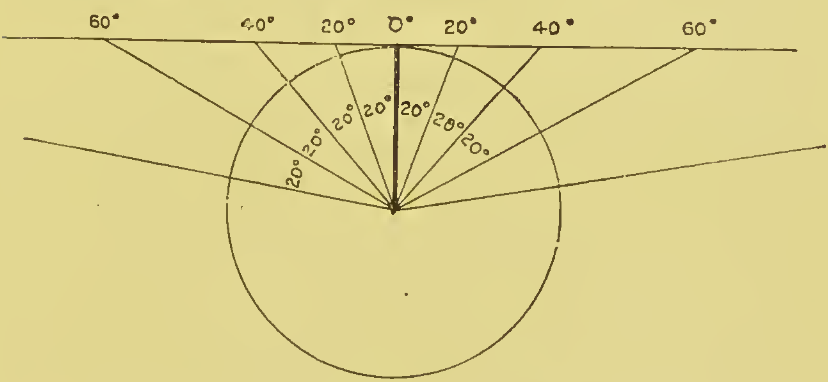

Fig. 68.—To illustrate a tangent scale.

the *arc*, and the greater the angle, the greater is the disproportion between the three quantities. *Fig.* 68 shews how when angles are projected on to

a straight line, as in the tangent scales represented in *Figs.* 59 and 60, the graduations increase in magnitude with each departure from zero. The following table (Table X.) of natural sines and tangents is inserted only for convenience, to save reference to a book of mathematical tables which may not always be at hand. The column of sines may be used for resultant prisms, and the column of tangents for making a tangent scale. For accurate working out of formulæ a fuller table must of course be used.

3.—*Formulæ for refraction through prisms.* In *Fig.* 69, a ray of light (ef) is seen traversing a

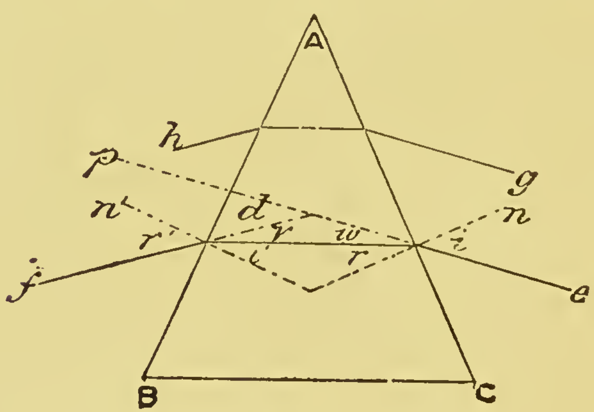

Fig. 69.

prism in the plane of a principal section. Then, whatever be the angle of incidence (i), the total deviation (d) is the sum of the deviations at the two surfaces, so that

$$(1) \qquad d = w + v.$$

TABLE X. (SINES AND TANGENTS.)

Degrees.	Sines.	Tangents.	Degrees.	Sines.	Tangents.
1°	·01745	·01745	23°	·3907	·4245
2°	·0349	·03492	24°	·4067	·4452
3°	·05233	·0524	25°	·4226	·4663
4°	·06975	·0699	26°	·4384	·4877
5°	·08715	·0874	27°	·4540	·5095
6°	·10452	·1051	28°	·4695	·5317
7°	·12187	·1228	29°	·4848	·5543
8°	·13917	·1405	30°	·5	·5773
9°	·15643	·1584	31°	·5150	·6009
10°	·17364	·1763	32°	·5299	·6249
11°	·19081	·1944	33°	·5446	·6494
12°	·2079	·2125	34°	·5592	·6745
13°	·2249	·2309	35°	·5736	·7002
14°	·2419	·2493	36°	·5878	·7265
15°	·2588	·2679	37°	·6018	·7535
16°	·2756	·2867	38°	·6157	·7813
17°	·2924	·3057	39°	·6293	·8098
18°	·3090	·3249	40°	·6428	·8390
19°	·3256	·3443	41°	·6560	·8693
20°	·3420	·3640	42°	·6691	·9004
21°	·3584	·3839	43°	·6820	·9325
22°	·3746	·4040	44°	·6946	·9657

Moreover, since n and n' are normals to the refracting surfaces,

$$(2) \qquad A = r + i'.$$

But
$$w = i - r.$$
and
$$v = r' - i'.$$
so that by substitution,

$$(3) \quad d = i + r' - A.$$

If the ray pass in the direction of *minimum deviation*, so that $i = r'$, then from (3)

$$(4) \qquad d = 2i - A.$$

and
$$(5) \qquad i = \frac{d + A}{2}.$$

This last formula tells us that if we wish to pass a ray through a prism in the direction of minimum deviation, we must allow an angle of incidence equal to half the sum of the deviating and refracting angles.

Moreover, under the same conditions,

$$r = \frac{A}{2}$$

and since, by the law of sines, if we let μ stand for the index of refraction,

$$\sin. i = \mu \sin. r.$$

$$(6) \qquad i = \sin.^{-1}\left(\mu \sin. \frac{A}{2}\right) - A.$$

Substituting this value for i in formula 4 we get

$$(7) \qquad d = 2 \sin.^{-1}\left(\mu \sin. \frac{A}{2}\right) - A.$$

This formula enables us to find the deviating

angle of any prism, given its refracting angle (A) and its index of refraction (μ).

If we treat angles and their sines as equivalents, which we may do without serious error in the case of very weak prisms, then

$$(8) \qquad d = A\,(\mu - 1).$$

This useful formula has been utilised on p. 19.

Lastly, if we wish to find the index of refraction of any transparent material, we can do so as follows.

By the law of sines,

$$(9) \qquad \mu = \frac{\sin.\ i}{\sin.\ r}.$$ And since

$$\text{(by 5)} \qquad i = \frac{d + A}{2}, \text{ and } r = \frac{A}{2},$$

$$(10) \qquad \mu = \frac{\sin.\ \dfrac{A + d}{2}}{\sin.\ \dfrac{A}{2}}$$

From this formula, after finding the refracting angle (A) of any prismatic piece of glass, and the deviating angle (d) by the methods described on pp. 3 and 30, we can readily discover the index of refraction.

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